National 5 Mathematics

Course Notes & Solutions

These notes are designed as a revision aid to the National 5 Mathematics course, they divide the course into 90 individual lessons. This document should link directly to the PowerPoint 'National 5 Course' as well as being backed up by the PowerPoints 'National 5 Exam Questions – by Topic', 'National 5 Exam Questions – by Paper' and 'National 5 Starter Questions'.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \frac{1}{2}ab \sin C$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3}Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n - 1}}$, where n is the sample size.

National 5 Learning Intentions

Unit One

1. Fractions & Percentages (9 Lessons)			
1.1 Appreciation	(a) I can round to a given number of significant figures.	5	1 2 3 4 5
	(b) I can carry out calculations involving appreciation and depreciation.	5	1 2 3 4 5
	(c) I can work with more challenging appreciation questions**.**	5	1 2 3 4 5
1.2 Reversing Changes	(a) I can reverse a percentage change to find an original quantity.	6	1 2 3 4 5
1.3 Fractions	(a) I can convert between top heavy-fractions and mixed numbers.	6	1 2 3 4 5
	(b) I can multiply two fractions together.	6	1 2 3 4 5
	(c) I can divide one fraction by another.	7	1 2 3 4 5
	(d) I can add and subtract two fractions.	7	1 2 3 4 5
	(e) I can work with fractions involving BIDMAS.	8	1 2 3 4 5

2. Geometric Skills (6 Lessons)			
2.1 Angle Properties	(a) I can calculate the interior angles of a regular polygon.	9	1 2 3 4 5
	(b) I can identify and use angle properties in circles.	9	1 2 3 4 5
2.2 Pythagoras' Uses	(a) I can use the Converse of Pythagoras' Theorem.	9	1 2 3 4 5
	(b) I can use Pythagoras' Theorem to solve problems in circles.	10	1 2 3 4 5
2.3 Similarity	(a) I can find a scale factor and use this to calculate a missing side.	11	1 2 3 4 5
	(b) I can work with area and volume scale factors.	11	1 2 3 4 5

3. Algebraic Operations (9 Lessons)			
3.1 Multiply out Brackets	(a) I can multiply out single brackets.	12	1 2 3 4 5
	(b) I can multiply out double (2 x 2) brackets.	12	1 2 3 4 5
	(c) I can multiply out double (2 x 3) brackets.	12	1 2 3 4 5
3.2 Factorising	(a) I can factorise by removing a common factor.	12	1 2 3 4 5
	(b) I can factorise the difference of two squares.	13	1 2 3 4 5
	(c) I can factorise a basic trinomial.	13	1 2 3 4 5
	(d) I can factorise a more complex trinomial.	13	1 2 3 4 5
3.3 Function Notation	(a) I can use function notation and substitution.	13	1 2 3 4 5
	(b) I can use function notation and equations.	13	1 2 3 4 5

4. Volume & Sectors (7 Lessons)			
4.1 Volume	(a) I can calculate the volume of basic three-dimensional shapes.	14	1 2 3 4 5
	(b) I can use the volume of a shape to calculate its height.	14	1 2 3 4 5
	(c) I can calculate the volume of composite shapes.	15	1 2 3 4 5
4.2 Arcs & Sectors	(a) I can calculate the arc length of a sector.	15	1 2 3 4 5
	(b) I can calculate the area of a sector.	16	1 2 3 4 5
	(c) I can use the arc length to calculate the angle or radius.	16	1 2 3 4 5
4.3 Gradient	(a) I can calculate the gradient of a straight line.	16	1 2 3 4 5

Unit Two

5. Linear Equations (11 Lessons)			
5.1 Solving Equations	(a) I can solve equations involving brackets.	18	1 2 3 4 5
	(b) I can solve equations involving fractions.	18	1 2 3 4 5
	(c) I can solve inequations.	18	1 2 3 4 5
5.2 Straight Lines	(a) I can identify the equation of a straight line.	19	1 2 3 4 5
	(b) I can identify the gradient and y-intercept from an equation.	19	1 2 3 4 5
	(c) I can use substitution to identify coordinates.	20	1 2 3 4 5
5.3 Simultaneous Eqns	(a) I can solve simultaneous equations graphically.	20	1 2 3 4 5
	(b) I can solve simultaneous equations algebraically.	20	1 2 3 4 5
	(c) I can construct and solve simultaneous equations in context.	21	1 2 3 4 5
5.4 Rearranging	(a) I can rearrange using the four basic operations.	22	1 2 3 4 5
	(b) I can rearrange using squares and square roots.	22	1 2 3 4 5

6. Trigonometric Graphs (6 Lessons)			
6.1 Trig Graphs	(a) I can work with the sine, cosine and tangent graphs.	23	1 2 3 4 5
	(b) I can identify graphs using amplitudes and frequencies.	23	1 2 3 4 5
	(c) I can identify graphs using phase angles and vertical translations.	24	1 2 3 4 5
6.2 Trig Equations	(a) I can solve basic trigonometric equations.	24	1 2 3 4 5
	(b) I can rearrange and solve trigonometric equations.	24	1 2 3 4 5
	(c) I can use trigonometric identities to simplify and prove statements.	25	1 2 3 4 5

7. Statistical Skills (4 Lessons)			
7.1 Standard Deviation	(a) I can calculate the mean and standard deviation of a data set.	26	1 2 3 4 5
7.2 Quartiles	(a) I can find the quartiles of a data set.	27	1 2 3 4 5
	(b) I can find the interquartile and semi-interquartile ranges.	27	1 2 3 4 5
7.3 Scatter Graphs	(a) I can find and use the equation of a scatter graph's line of best fit.	27	1 2 3 4 5

8. Quadratics (11 Lessons)			
8.1 Completed Square	(a) I can write an expression in the completed square form.	29	1 2 3 4 5
	(b) I can identify the equation of a parabola in completed square form.	29	1 2 3 4 5
	(c) I can identify the symmetry and turning point of a parabola.	29	1 2 3 4 5
	(d) I can work with quadratics and substitution.	29	1 2 3 4 5
	(e) I can sketch a quadratic graph in completed square form.	30	1 2 3 4 5
8.2 Factorised Form	(a) I can sketch a quadratic graph in factorised form.	30	1 2 3 4 5
	(b) I can find the roots of a quadratic equation using factorisation.	31	1 2 3 4 5
	(c) I can rearrange, factorise and solve quadratic equations.	31	1 2 3 4 5
	(d) I can solve quadratic equations in context.	32	1 2 3 4 5
8.3 Quadratic Formula	(a) I can use the quadratic formula to solve problems.	32	1 2 3 4 5
	(b) I can use the discriminant to determine the nature of roots.	33	1 2 3 4 5

Unit Three

9. Surds & Indices (8 Lessons)			
9.1 Surds	(a) I can simplify basic surds.	34	1 2 3 4 5
	(b) I can simplify surds by multiplying and dividing.	34	1 2 3 4 5
	(c) I can simplify surds by adding and subtracting.	34	1 2 3 4 5
	(d) I can rationalise the denominator of a fraction.	34	1 2 3 4 5
9.2 Indices	(a) I can solve problems involving the basic rules of indices.	35	1 2 3 4 5
	(b) I can solve problems involving negative indices.	35	1 2 3 4 5
	(c) I can solve problems involving fractional indices.	36	1 2 3 4 5
9.3 Scientific Notation	(a) I can carry out calculations involving scientific notation.	36	1 2 3 4 5

10. Vectors (6 Lessons)			
10.1 Vector Calculations	(a) I can convert between directed line segments and column vectors.	37	1 2 3 4 5
	(b) I can add directed line segments and multiply by scalars.	37	1 2 3 4 5
	(c) I can add column vectors and multiply by scalars.	38	1 2 3 4 5
10.2 Magnitude	(a) I can calculate the magnitude of a vector.	38	1 2 3 4 5
10.3 Vector Pathways	(a) I can identify coordinates in three dimensions.	39	1 2 3 4 5
	(b) I can work with vector pathways.	39	1 2 3 4 5

11. Trigonometric Skills (9 Lessons)			
11.1 The Area Rule	(a) I can use the area rule to calculate the area of a triangle.	40	1 2 3 4 5
	(b) I can use the area rule to calculate the area of a composite shape.	40	1 2 3 4 5
11.2 The Sine Rule	(a) I can use the sine rule to calculate a missing side.	40	1 2 3 4 5
	(b) I can use the sine rule to calculate a missing angle.	41	1 2 3 4 5
	(c) I can use trigonometry along with the sine rule.	41	1 2 3 4 5
11.3 The Cosine Rule	(a) I can use the cosine rule to calculate a missing side.	42	1 2 3 4 5
	(b) I can use the cosine rule to calculate a missing angle.	42	1 2 3 4 5
	(c) I can use trigonometry along with the cosine rule.	43	1 2 3 4 5
11.4 Bearings	(a) I can use bearings to solve problems.	43	1 2 3 4 5

12. Algebraic Fractions (4 Lessons)			
12.1 Simplifying Fractions	(a) I can factorise and simplify algebraic fractions.	45	1 2 3 4 5
12.2 Algebraic Fractions	(a) I can multiply two algebraic fractions.	45	1 2 3 4 5
	(b) I can divide one algebraic fraction by another.	45	1 2 3 4 5
	(c) I can add and subtract two algebraic fractions.	45	1 2 3 4 5

1. Fractions & Percentages

1.1 Appreciation

1.1 (a) I can round to a given number of significant figures.

You should be able to round a given number to a required number of **significant figures**.

Number	7 462	59 357	43.573	2.86521	0.024 781	0.007 438	0.000 262 8
1 sig. fig.	7 000	60 000	40	3	0.02	0.007	0.000 3
2 sig. fig.	7 500	59 000	44	2.9	0.025	0.007 4	0.000 26
3 sig. fig.	7 460	59 400	43.6	2.87	0.024 8	0.007 44	0.000 263

1.1 (b) I can carry out calculations involving appreciation and depreciation.

Appreciation means an **increase** in size. **Depreciation** means a **decrease** in size.

You can calculate a **percentage increase** or **decrease** in one simple step using a **multiplier.**

	Increase						**Decrease**					
Percentage	2%	5%	10%	16%	30%	1.4%	4%	9%	10%	18%	40%	2.7%
Multiplier	1.02	1.05	1.1	1.16	1.3	1.014	0.96	0.91	0.9	0.82	0.6	0.973

You can solve problems on **appreciation** and **depreciation** using **multipliers** and **powers**.

Kirsty runs 18 miles per week. She decides to increase her distance by 10% a week. How many miles will she run in week 4? $18 \times 1.1^4 = 26.35\ miles$	A company wants to reduce the sugar content in its cereal by 3% a year. If the sugar content is currently 35 g what will it be in 7 years? $35 \times 0.97^7 = 28.28\ g$	One evening the temperature drops 10.2% per hour. At 6 pm the temperature was 27°C. Find the temperature at 11 pm. $27 \times 0.898^5 = 15.77^o C$	The population of a town, Alness, is growing by 5% annually. If the population is currently 6 000 what will it be in 2 years. $6\,000 \times 1.05^2 = 6\,615$

1.1 (c) I can work with more challenging appreciation questions.

You may be given a question on **appreciation** or **depreciation** requiring some more thought.

Brenda weighs 80 kg on January the 1st. She plans to reduce her weight by 3% a month. In which month will her weight first drop below 72 kg? $1\ F: 80 \times 0.97^1 = 77.6\ kg$ $1\ M: 80 \times 0.97^2 = 75.27\ kg$ $1\ A: 80 \times 0.97^3 = 73.01\ kg$ $1\ M: 80 \times 0.97^4 = 70.82\ kg$ *She will reach her target during April.*	A bus is worth £20 000 it will be sold when its value is less than £12 000. If its value depreciates by 10% a year, when will it be sold? $1: 20\,000 \times 0.9^1 = £18\,000$ $2: 20\,000 \times 0.9^2 = £16\,200$ $3: 20\,000 \times 0.9^3 = £14\,580$ $4: 20\,000 \times 0.9^4 = £13\,122$ $5: 20\,000 \times 0.9^5 = £11\,809.80$ *It will be sold after 5 years.*	The population of a city is rising by 20% a year. If its population is 200 000 after how many years will its population exceed 400 000? $1: 200\,000 \times 1.2^1 = 240\,000$ $2: 200\,000 \times 1.2^2 = 288\,000$ $3: 200\,000 \times 1.2^3 = 345\,600$ $4: 200\,000 \times 1.2^4 = 414\,720$ *It will exceed this after 4 years.*

1.2 Reversing Changes

1.2 (a) I can reverse a percentage change to find an original quantity.

When trying to calculate the original value of something, **before a percentage change**, you **divide by the multiplier** instead of multiplying by it.

Andy gets a 5% pay rise. After the pay rise, he is earning £2 604 a month. What was he earning before his pay rise? $2\,604 \div 1.05 = £2\,480$	A car is valued at £3 780. This is 16% less than it was valued at last year. What was the value of the car last year? $3\,780 \div 0.84 = £4\,500$	54 000 tickets were sold for a concert. This was 90% of those available. Calculate the number of tickets available. $54\,000 \div 0.9 = 60\,000$	At the end of a year a tree is 330 cm tall. If it has grown by 10% in the last year what was its height at the beginning? $330 \div 1.1 = 300\ cm$

1.3 Fractions

1.3 (a) I can convert between top heavy-fractions and mixed numbers.

A **top-heavy fraction** is one where the numerator is greater than the denominator.

You should be able to change a **top-heavy fraction** to a **mixed number**.

$\frac{12}{5}$	$\frac{25}{6}$	$\frac{30}{7}$	$\frac{50}{9}$	$\frac{32}{5}$	$\frac{44}{7}$	$\frac{47}{9}$	$\frac{19}{5}$	$\frac{55}{8}$	$\frac{35}{6}$
$2\frac{2}{5}$	$4\frac{1}{6}$	$4\frac{2}{7}$	$5\frac{5}{9}$	$6\frac{2}{5}$	$6\frac{2}{7}$	$5\frac{2}{9}$	$3\frac{4}{5}$	$6\frac{7}{8}$	$5\frac{5}{6}$

You should also be able to work the other way around and change a **mixed number** to a **top-heavy fraction**.

$2\frac{1}{3}$	$4\frac{3}{5}$	$7\frac{3}{4}$	$3\frac{2}{9}$	$1\frac{3}{10}$	$2\frac{2}{7}$	$1\frac{7}{9}$	$5\frac{4}{5}$	$2\frac{7}{8}$	$9\frac{5}{6}$
$\frac{7}{3}$	$\frac{23}{5}$	$\frac{31}{4}$	$\frac{29}{5}$	$\frac{13}{10}$	$\frac{16}{7}$	$\frac{16}{9}$	$\frac{29}{5}$	$\frac{23}{8}$	$\frac{59}{6}$

1.3 (b) I can multiply two fractions together.

To **multiply** two fractions together:

1. Make sure to convert any **mixed number** to a **top-heavy fraction**.
2. **Simplify** by checking along the **diagonals**.
3. Multiply the **two numerators** and the **two denominators**.
4. If this is a **top-heavy fraction** convert it to a **mixed number**.

$\frac{2}{5} \times \frac{3}{4}$. $= \frac{\cancel{2}^{1}}{5} \times \frac{3}{\cancel{4}_{2}}$ $= \frac{3}{10}$	$\frac{13}{3} \times \frac{6}{7}$. $= \frac{13}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{7}$ $= \frac{26}{7}$ $= 3\frac{5}{7}$	$1\frac{4}{5} \times \frac{2}{3}$. $= \frac{\cancel{9}^{3}}{5} \times \frac{2}{\cancel{3}_{1}}$ $= \frac{6}{5}$ $= 1\frac{1}{5}$	$3\frac{3}{4} \times 2\frac{1}{5}$. $= \frac{\cancel{15}^{3}}{4} \times \frac{11}{\cancel{5}_{1}}$ $= \frac{33}{4}$ $= 8\frac{1}{4}$	$1\frac{1}{6} \times 3\frac{1}{3}$. $= \frac{7}{\cancel{6}_{3}} \times \frac{\cancel{10}^{5}}{3}$ $= \frac{35}{9}$ $= 3\frac{8}{9}$

1.3 (c) I can divide one fraction by another.

To **divide** one fraction by another:

1. Make sure to convert any **mixed number** to a **top-heavy fraction**.
2. **Flip** the second fraction upside down and **multiply** the two fractions.
3. If this is a **top-heavy fraction** convert it to a **mixed number**.

$\frac{9}{4} \div \frac{7}{2}$.	$\frac{3}{8} \div \frac{5}{16}$.	$4\frac{1}{3} \div \frac{5}{6}$.	$2\frac{2}{7} \div 1\frac{1}{3}$.	$3\frac{1}{2} \div \frac{3}{8}$.
$= \frac{9}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{7}$	$= \frac{\cancel{3}_{1}}{\cancel{8}} \times \frac{\cancel{16}^{2}}{5}$	$= \frac{13}{3} \div \frac{5}{6}$	$= \frac{16}{7} \div \frac{4}{3}$	$= \frac{7}{2} \div \frac{3}{8}$
$= \frac{9}{14}$	$= \frac{6}{5}$	$= \frac{13}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{5}$	$= \frac{\cancel{16}^{4}}{7} \times \frac{3}{\cancel{4}_{1}}$	$= \frac{7}{\cancel{2}_{1}} \times \frac{\cancel{8}^{4}}{3}$
	$= 1\frac{1}{5}$	$= \frac{26}{5}$	$= \frac{12}{7}$	$= \frac{28}{3}$
		$= 5\frac{1}{5}$	$= 1\frac{5}{7}$	$= 9\frac{1}{3}$

1.3 (d) I can add and subtract two fractions.

To **add** or **subtract** two fractions:

1. If you have any **mixed numbers** you can covert these to **top-heavy fractions**.
2. Make sure the **denominators** are the **same**.
3. Add or subtract the two **numerators**.
4. If this is a **top-heavy fraction** convert it to a **mixed number.**
5. Make sure to **simplify** the fraction if possible.

$\frac{5}{8} - \frac{1}{3}$.	$\frac{9}{4} + \frac{5}{6}$.	$3\frac{3}{4} + 2\frac{1}{3}$.	$5\frac{1}{4} + 1\frac{1}{8}$.	$4\frac{2}{3} - 2\frac{1}{6}$.
$= \frac{15}{24} - \frac{8}{24}$	$= \frac{27}{12} + \frac{10}{12}$	$= \frac{15}{4} + \frac{7}{3}$	$= \frac{21}{4} + \frac{9}{8}$	$= \frac{14}{3} - \frac{13}{6}$
$= \frac{7}{24}$	$= \frac{37}{12}$	$= \frac{45}{12} + \frac{28}{12}$	$= \frac{42}{8} + \frac{9}{8}$	$= \frac{28}{6} - \frac{13}{6}$
	$= 3\frac{1}{12}$	$= \frac{73}{12}$	$= \frac{51}{8}$	$= \frac{15}{6}$
		$= 6\frac{1}{12}$	$= 6\frac{3}{8}$	$= 2\frac{3}{6}$
				$= 2\frac{1}{2}$

1.3 (e) I can work with fractions involving BIDMAS.

From National 4: You should know the following:

BIDMAS is an acronym used to help you remember the **order of operations**.

BRACKETS INDICES DIVISION MULTIPLICATION ADDITION SUBTRACTION

In National 5: You should be able to use **BIDMAS** with **fractions**.

Note: When the word '**of**' is used this just means **multiply**.

$\frac{3}{5}\left(\frac{3}{4}-\frac{2}{9}\right)$.

$=\frac{3}{5}\times\left(\frac{27}{36}-\frac{8}{36}\right)$

$=\frac{\cancel{3}^{1}}{5}\times\frac{19}{\cancel{36}_{12}}$

$=\frac{19}{60}$

$\frac{2}{3}\left(1\frac{1}{3}-\frac{1}{2}\right)$.

$=\frac{2}{3}\times\left(\frac{4}{3}-\frac{1}{2}\right)$

$=\frac{2}{3}\times\left(\frac{8}{6}-\frac{3}{6}\right)$

$=\frac{\cancel{2}^{1}}{3}\times\frac{5}{\cancel{6}_{3}}$

$=\frac{5}{9}$

$6\frac{2}{3}$ of $\left(\frac{4}{5}-\frac{3}{4}\right)$.

$=\frac{20}{3}\times\left(\frac{4}{5}-\frac{3}{4}\right)$

$=\frac{20}{3}\times\left(\frac{16}{20}-\frac{15}{20}\right)$

$=\frac{\cancel{20}^{1}}{3}\times\frac{1}{\cancel{20}_{1}}$

$=\frac{1}{3}$

$\frac{2}{3}$ of $\left(1\frac{1}{3}+\frac{3}{4}\right)$.

$=\frac{2}{3}\times\left(\frac{4}{3}+\frac{3}{4}\right)$

$=\frac{2}{3}\times\left(\frac{16}{12}+\frac{9}{12}\right)$

$=\frac{\cancel{2}^{1}}{3}\times\frac{25}{\cancel{12}_{6}}$

$=\frac{25}{18}$

$=1\frac{7}{18}$

$1\frac{1}{2}-\frac{1}{6}\div\frac{2}{3}$.

$=\frac{3}{2}-\frac{1}{6}\div\frac{2}{3}$

$=\frac{3}{2}-\frac{1}{\cancel{6}_{2}}\times\frac{\cancel{3}^{1}}{2}$

$=\frac{3}{2}-\frac{1}{4}$

$=\frac{6}{4}-\frac{1}{4}$

$=\frac{5}{4}$

$=1\frac{1}{4}$

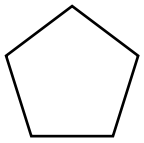
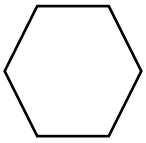
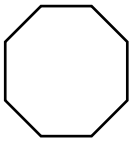
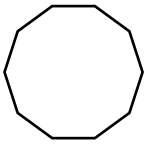
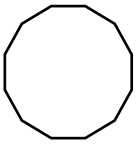
2. Geometric Skills

2.1 Angle Properties

2.1 (a) I can calculate the interior angles of a regular polygon.

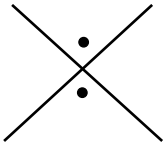
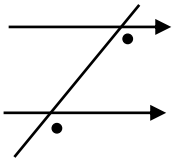
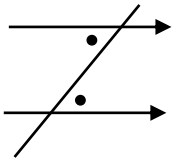
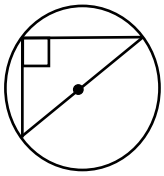
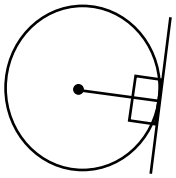
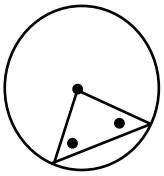
A **polygon** is a two-dimensional shape with straight sides. The sum of the angles of a triangle is 180°.

A **regular polygon** has **equal sides** and **angles**. You can calculate the **interior angles** of a regular polygon.

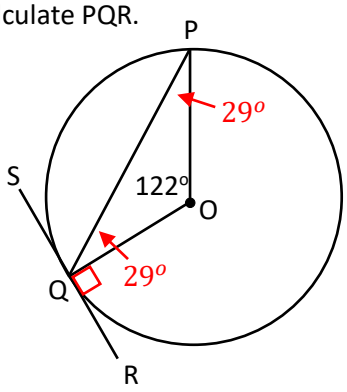
				
$360^o \div 5 = 72^o$ $180^o - 72^o = 108^o$	$360^o \div 6 = 60^o$ $180^o - 60^o = 120^o$	$360^o \div 8 = 45^o$ $180^o - 45^o = 135^o$	$360^o \div 10 = 36^o$ $180^o - 36^o = 144^o$	$360^o \div 12 = 30^o$ $180^o - 30^o = 150^o$

2.1 (b) I can identify and use angle properties in circles.

You should know the following **angle properties** and be able to use these to solve problems:

					
Opposite angles	Corresponding (F) angles	Alternate (Z) angles	Right angle in a semi-circle	Tangent to circle	Isosceles triangle in circle

Calculate PQR.

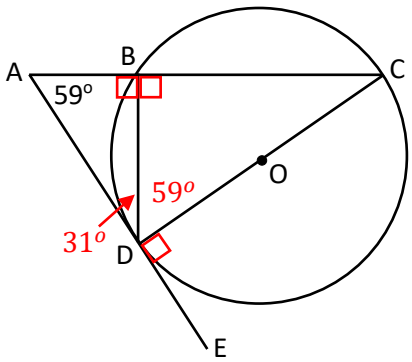


$OQR = 90^o$

$OQP = (180^o - 122^o) \div 2 = 29^o$

$PQR = 90^o + 29^o = 119^o$

Calculate BDE.



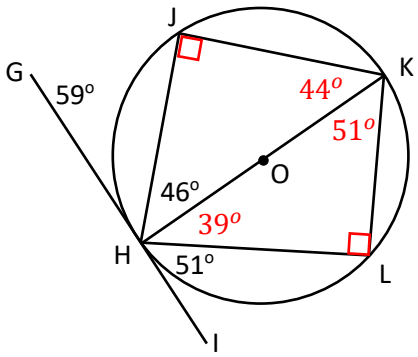
$CBD = ABD = CDE = 90^o$

$ADB = 180^o - 90^o - 59^o = 31^o$

$BDC = 90^o - 39^o = 59^o$

$BDE = 59^o + 90^o = 149^o$

Calculate JKL.



$HLK = HJK = 90^o$

$KHL = 90^o - 51^o = 39^o$

$HKL = 180^o - 90^o - 39^o = 51^o$

$HKJ = 180^o - 90^o - 46^o = 44^o$

$JKL = 51^o + 44^o = 95^o$

2.2 Pythagoras' Uses

2.2 (a) I can use the Converse of Pythagoras' Theorem.

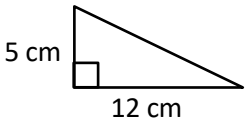
From National 4: You should know the following:

Pythagoras' theorem states that for all **right-angled triangles**, $c^2 = a^2 + b^2$, alternatively $b^2 = c^2 - a^2$.

In National 5: You should also be able to use the **Converse of Pythagoras' Theorem** which states:

If in a triangle ABC, $\boldsymbol{c^2 = a^2 + b^2}$, then the triangle is **right-angled** at C.

Calculate the missing side.



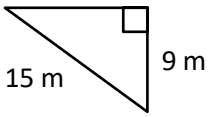
$c^2 = 5^2 + 12^2$

$= 169$

$c = \sqrt{169}$

$= 13\ cm$

Calculate the missing side.



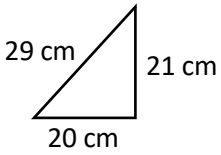
$b^2 = 15^2 - 9^2$

$= 144$

$b = \sqrt{144}$

$= 12\ m$

Prove the triangle is right angled.

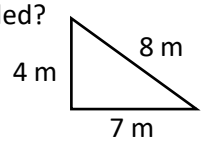


$29^2 = 841$

$21^2 + 20^2 = 841$

By the converse of Pythagoras, as $29^2 = 21^2 + 20^2$ *the triangle is right angled.*

Is this triangle right angled?



$8^2 = 64$

$4^2 + 7^2 = 65$

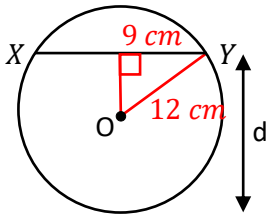
By the converse of Pythagoras, as $8^2 \neq 4^2 + 7^2$ *the triangle is not right angled.*

2.2 (b) I can use Pythagoras' Theorem to solve problems in circles.

You can combine **Pythagoras' Theorem** and **Angle Properties** to solve problems in circles:

1. Create a **right-angled triangle** using the given information.
2. Use **Pythagoras' Theorem**: $b^2 = c^2 - a^2$.
3. Take the **square root**.
4. Use the required information to solve the problem.

The diagram shows the cross-section of a pipe.



The radius of the pipe is 12 cm. XY is 18 cm

Calculate the depth d.

$b^2 = 12^2 - 9^2$

$= 63$

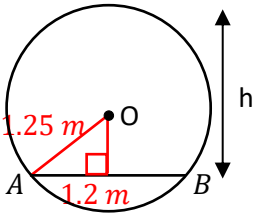
$b = \sqrt{63}$

$= 7.94\ cm$

$d = 12 + 7.94$

$= 59.94\ cm$

The diagram shows the cross-section of a tunnel.



The radius of the tunnel is 1.25 m. AB is 2.4 m long.

Calculate the height h.

$b^2 = 1.25^2 - 1.2^2$

$= 0.1225$

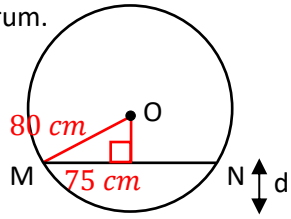
$b = \sqrt{0.1225}$

$= 0.35\ m$

$h = 1.25 + 0.35$

$= 1.6\ m$

The diagram shows the cross-section of an oil drum.



The radius of the drum is 80 cm. MN is 150 cm long.

Calculate the depth d.

$b^2 = 80^2 - 75^2$

$= 775$

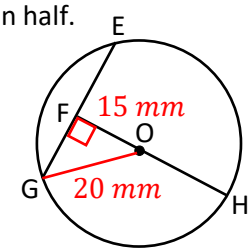
$b = \sqrt{775}$

$= 27.84\ cm$

$d = 80 - 27.84$

$= 52.16\ cm$

The point F splits the line EG in half.



The radius of the circle is 20 mm. FH is 35 cm long.

Calculate the size of EG.

$b^2 = 20^2 - 15^2$

$= 175$

$b = \sqrt{175}$

$= 13.23\ mm$

$EG = 13.23 \times 2$

$= 26.46\ mm$

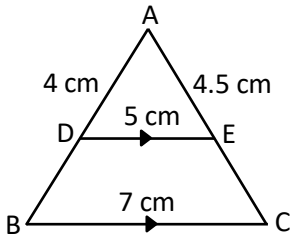
2.3 Similarity

2.3 (a) I can find a scale factor and use this to calculate a missing side.

Two figures are **similar** when they are the **same shape** but **different sizes**. The difference in their sizes is known as the **scale factor.** To solve questions involving **scale factors**:

1. Find two **corresponding sides** which both have measurements given.
2. Divide one by the other to produce the **scale factor**.
3. Find the side that corresponds to the one you are trying to find.
4. Multiply this side by the **scale factor**.

In the diagram DE is parallel to BC.



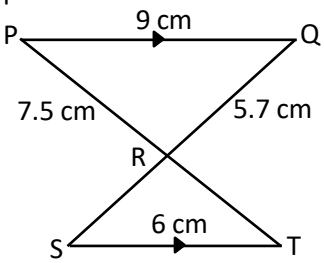
Calculate the size of AB.

$SF = \frac{7}{5}$

$AB = \frac{7}{5} \times 4$

$= 6.6\ cm$

In the diagram PQ is parallel to ST.



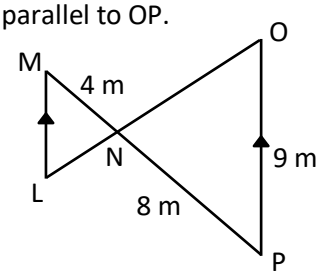
Calculate the length of RS.

$SF = \frac{6}{9} = \frac{2}{3}$

$RS = \frac{2}{3} \times 5.7$

$= 3.8\ cm$

In the diagram ML is parallel to OP.



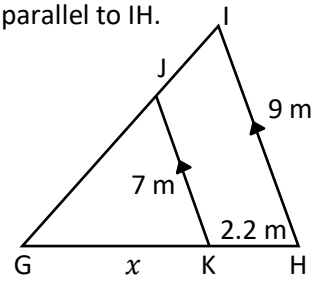
Calculate the size of ML.

$SF = \frac{4}{8} = \frac{1}{2}$

$RS = \frac{1}{2} \times 9$

$= 4.5\ m$

In the diagram JK is parallel to IH.



Calculate the length of x.

$SF = \frac{7}{9}$

$x = \frac{7}{9}(x + 2.2)$

$9x = 7(x + 2.2)$

$9x = 7x + 15.4$

$2x = 15.4$

$x = 7.7\ m$

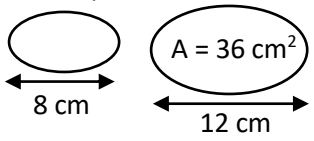
2.3 (b) I can work with area and volume scale factors.

If two figures are **similar** and their **scale factor** is k:

The **area scale factor** is k^2.

The **volume scale factor** is k^3.

Two ellipses are similar.



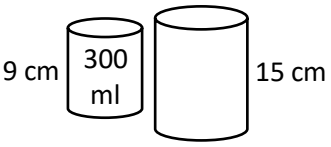
Find the area of the smaller ellipse.

$SF = \frac{8}{12} = \frac{2}{3}$

$Area = \left(\frac{2}{3}\right)^2 \times 36$

$= 16\ cm^2$

Two cans are similar.



Find the volume of the larger can.

$SF = \frac{15}{9} = \frac{5}{3}$

$Volume = \left(\frac{5}{3}\right)^3 \times 300$

$= 1\ 388.89\ ml$

Two pictures are similar in shape. The large one is 80 cm high and has an area of 5 600 cm^2. The small one is 12 cm high.

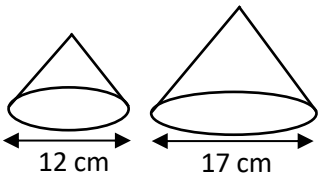
What will its area be?

$SF = \frac{12}{80} = \frac{3}{20}$

$Area = \left(\frac{3}{20}\right)^2 \times 5\ 600$

$= 126\ cm^2$

Are these cones similar?



V = 400 ml V = 1130 ml

$SF = \frac{17}{12}$

$Volume = \left(\frac{17}{12}\right)^3 \times 400$

$= 1\ 137.27\ ml$

$Therefore\ not\ similar\ as\ 1137.27 \neq 1130\ ml.$

3. Algebraic Operations

3.1 Multiply out Brackets

3.1 (a) I can multiply out single brackets.

You are required to **multiply out single brackets** and gather like terms – a **grid** can help in this.

$2y(3x - y)$

$= 6xy - 2y^2$

$m + 3(2m - 5)$

$= m + 6m - 15$

$= 7m - 15$

$c(2c + 1) + 4c$

$= 2c^2 + c + 4c$

$= 2c^2 + 5c$

$3(d + 6) - 2(2d - 1)$

$= 3d + 18 - 4d + 2$

$= -d + 20$

3.1 (b) I can multiply out double (2 x 2) brackets.

You are also required to **multiply out double (2 x 2) brackets** and gather like terms – a **grid** can again help in this.

$(4m + 5)(2m + 3)$

	$2m$	$+3$
$4m$	$8m^2$	$+12m$
$+5$	$+10m$	$+15$

$8m^2 + 22m + 15$

$(2x - 1)(x + 7)$

	x	$+7$
$2x$	$2x^2$	$+14x$
-1	$-x$	-7

$2x^2 + 13x - 7$

$(2y - 3)^2$

	$2y$	-3
$2y$	$4y^2$	$-6y$
-3	$-6y$	$+9$

$4y^2 - 12y + 9$

$2 + (3x + 2)(x - 4)$

	x	-4
$3x$	$3x^2$	$-12x$
$+2$	$+2x$	-8

$2 + 3x^2 - 10x - 8$

$= 3x^2 - 10x - 6$

3.1 (c) I can multiply out double (2 x 3) brackets.

Lastly you are required to **multiply out double (2 x 3) brackets** and gather like terms – a **grid** can again help in this.

$(y + 4)(2y^2 + 5y + 7)$

	$2y^2$	$+5y$	$+7$
y	$2y^3$	$+5y^2$	$+7y$
$+4$	$+8y^2$	$+20y$	$+28$

$2y^3 + 13y^2 + 27y + 28$

$(x + 2)(x^2 - 2x + 5)$

	x^2	$-2x$	$+5$
x	x^3	$-2x^2$	$+5x$
$+2$	$+2x^2$	$-4x$	$+10$

$x^3 + x + 10$

$(d^2 - 4d - 2)(3d - 1)$

	d^2	$-4d$	-2
$3d$	$3d^3$	$-12d^2$	$-6d$
-1	$-d^2$	$+4d$	$+2$

$3d^3 - 13d^2 - 2d + 2$

3.2 Factorising

3.2 (a) I can factorise by removing a common factor.

Factorising is the opposite process of **multiplying out brackets**.

The first thing you should **ALWAYS** do when factorising is look for a **common factor** – a **grid** can help in this.

This **common factor** could be a (i) **number** (ii) **letter** or (iii) **combination** of the two.

$7k - 21$

$= 7(k - 3)$

$12a - 8b$

$= 4(3a - 2b)$

$8m - m^2$

$= m(8 - m)$

$6mn - 2n^2$

$= 2n(3m - n)$

$4xy - y^2$

$= y(4x - y)$

$2cd - 8cd^2$

$= 2cd(1 - 4d)$

3.2 (b) I can factorise the difference of two squares.

You are expected to be able to factorise the **difference of two squares**. **REMEMBER** to look for a **common factor** first.

$64 - 9k^2$ $= (8 + 3k)(8 - 3k)$	$25x^2 - 16y^2$ $= (5x + 4y)(5x - 4y)$	$4w^2 - 100$ $= 4(w^2 - 25)$ $= 4(w + 5)(w - 5)$	$18 - 32a^2$ $= 2(9 - 16a^2)$ $= 2(3 + 4a)(3 - 4a)$

3.2 (c) I can factorise a basic trinomial.

You are expected to be able to factorise **basic trinomials**. **REMEMBER** to look for a **common factor** first.

$x^2 + 8x + 15$ $= (x + 3)(x + 5)$	$y^2 - 15y + 56$ $= (y - 7)(y - 8)$	$2m^2 + 4m - 96$ $= 2(m^2 + 2n - 48)$ $= 2(m + 8)(m - 6)$	$3x^2 + 6x - 24$ $= 3(x^2 + 2x - 8)$ $= 3(x + 4)(x - 2)$

3.2 (d) I can factorise a more complex trinomial.

You are expected to be able to factorise **more complex trinomials**. **REMEMBER** to look for a **common factor** first.

$2x^2 + 7x + 3$ $= (x + 3)(2x + 1)$	$5m^2 - 3m - 2$ $= (m - 1)(5m + 2)$	$3y^2 - 4y - 4$ $= (y - 2)(3y + 2)$	$14d^2 - 30d + 4$ $= 2(7d^2 - 15d + 2)$ $= 2(d - 2)(7d - 1)$

3.3 Function Notation

3.3 (a) I can use function notation and substitution.

Function notation is a way of expressing the relationship between two variables. It can be used alongside **substitution** to find the value of a **function**.

If $f(x) = 5x - 12$ find $f(-2)$. $f(-2) = 5 \times (-2) - 12$ $= -22$	If $m(x) = 9 - 2x$ find $m(-5)$. $m(-5) = 9 - 2 \times (-5)$ $= 19$	If $g(x) = x^2 - 4x$ find $g(10)$. $g(10) = 10^2 - 4 \times 10$ $= 60$	If $a(x) = 10x^2$ find $a(-3)$. $a(-3) = 10 \times (-3)^2$ $= 90$

3.3 (b) I can use function notation and equations.

You can also use **function notation** to solve problems involving **equations**.

If $g(x) = 8 - 4x$ and $g(t) = -4$, find t. $8 - 4t = -4$ $-4t = -12$ $t = 3$	If $f(x) = 3x + 8$ and $f(a) = 101$, find a. $3a + 8 = 101$ $3a = 93$ $a = 31$	If $d(x) = 5x - 10$ and $d(t) = 25$, find t. $5t - 10 = 25$ $5t = 35$ $t = 7$	If $s(x) = 9 - 2x$ and $s(a) = -8$ find t. $9 - 2a = -8$ $-2a = -17$ $a = 8.5$

4. Volume & Sectors

4.1 Volume

4.1 (a) I can calculate the volume of basic three-dimensional shapes.

From National 4: You should know the following:

Area Formulae

$A_{rectangle} = L \times B$

$A_{triangle} = \frac{1}{2} \times b \times h$

$A_{circle} = \pi \times r^2$

Volume Formulae

$V_{cuboid} = L \times B \times H$

$V_{cylinder} = \pi \times r^2 \times h$

$V_{prism} = A \times h$

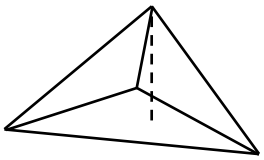
In National 5: You are expected to use three more formulae (given in the **exam formulae sheet**):

$V_{pyramid} = \frac{1}{3} \times A \times h$

$V_{cone} = \frac{1}{3} \times \pi \times r^2 \times h$

$V_{sphere} = \frac{4}{3} \times \pi \times r^3$

A pyramid has a base area 24 cm^2 and height 7 cm.

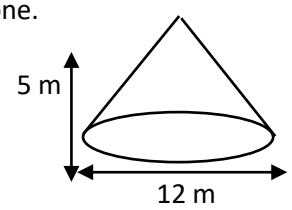


Find its volume.

$V_{pyramid} = \frac{1}{3} \times 24 \times 7$

$= 56\ cm^3$

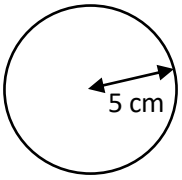
Find the volume of the cone.



$V_{cone} = \frac{1}{3} \times \pi \times 6^2 \times 5$

$= 188.5\ m^3$

Find the volume of the sphere.



$V_{sphere} = \frac{4}{3} \times \pi \times 5^3$

$= 523.6\ cm^3$

Using $\pi = 3.14$ find the volume of a cone with radius 5 m and height 3 m. (**non-calculator**)

$V_{cone} = \frac{1}{3} \times 3.14 \times 5^2 \times 3$

$= \frac{1}{3} \times 3.14 \times 75$

$= 25 \times 3.14$

$= 78.5\ cm^3$

4.1 (b) I can use the volume of a shape to calculate its height.

In some questions you may be given the **volume** of a **shape** and asked to calculate its **height**.

A cylinder has a volume of 2 300 m^3. The radius of the cross section is 8.2 m.

Calculate its' height.

$V_{cylinder} = \pi \times 8.2^2 \times h$

$2300 = \pi \times 8.2^2 \times h$

$\frac{2300}{\pi \times 8.2^2} = h$

$h = 10.89\ m$

A cone has a volume of 127 cm^3. The radius of the cross section is 4 cm.

Calculate its' height.

$V_{cone} = \frac{1}{3} \times \pi \times 4^2 \times h$

$127 = \frac{1}{3} \times \pi \times 4^2 \times h$

$127 \times 3 = \pi \times 4^2 \times h$

$\frac{127 \times 3}{\pi \times 4^2} = h$

$h = 5.59\ cm$

A square based pyramid has volume 324 m^3. The sides of the base are 9 m.

Calculate its' height.

$V_{pyramid} = \frac{1}{3} \times 81 \times h$

$324 = 27 \times h$

$\frac{324}{27} = h$

$h = 12\ m$

A cone has a volume of 5 890 cm^3. The radius of the cross section is 9 cm.

Calculate its' height.

$V_{cone} = \frac{1}{3} \times \pi \times 9^2 \times h$

$5\,890 = \frac{1}{3} \times \pi \times 9^2 \times h$

$5\,890 \times 3 = \pi \times 9^2 \times h$

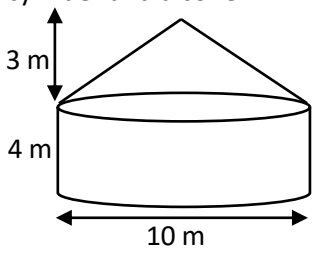
$\frac{5\,890 \times 3}{\pi \times 9^2} = h$

$h = 69.44\ cm$

4.1 (c) I can calculate the volume of composite shapes.

A **composite shape** is one made up of two or more standard shapes. You can calculate the volume of these.

A shape is made from a cylinder and a cone.



Calculate the volume of the shape, correct to **two significant figures**.

$V_{cone} = \frac{1}{3} \times \pi \times 5^2 \times 3$

$= 78.54\ m^3$

$V_{cylinder} = \pi \times 5^2 \times 4$

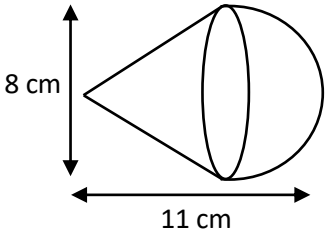
$= 314.16\ m^3$

$V_{total} = 78.54 + 314.16$

$= 392.70$

$= 390\ m^3$

A shape is made from a hemisphere and a cone.



Calculate the volume of the shape, correct to **one significant figure**.

$V_{cone} = \frac{1}{3} \times \pi \times 4^2 \times 7$

$= 117.29\ cm^3$

$V_{hemisphere} = \frac{2}{3} \times \pi \times 4^3$

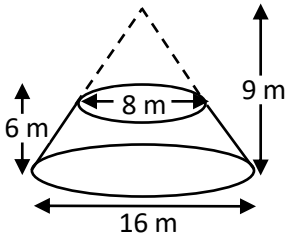
$= 134.04\ cm^3$

$V_{total} = 117.29 + 134.04$

$= 251.33$

$= 300\ cm^3$

A shape is made from a cone with its top cut off.



Calculate the volume of the shape, correct to **one decimal place.**

$V_{l\ cone} = \frac{1}{3} \times \pi \times 8^2 \times 9$

$= 603.19\ m^3$

$V_{s\ cone} = \frac{1}{3} \times \pi \times 4^2 \times 3$

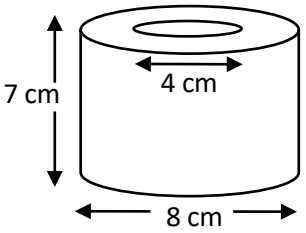
$= 50.27\ m^3$

$V_{total} = 603.19 - 50.27$

$= 552.92$

$= 552.9\ m^3$

The following shape has had a section removed.



Calculate the volume of the shape, correct to **two significant figures**.

$V_{l\ cylinder} = \pi \times 4^2 \times 7$

$= 351.86\ cm^3$

$V_{s\ cylinder} = \pi \times 2^2 \times 7$

$= 87.96\ m^3$

$V_{total} = 351.86 - 87.96$

$= 263.90$

$= 260\ m^3$

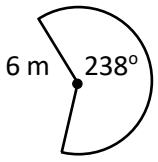
4.2 Arcs & Sectors

4.2 (a) I can calculate the arc length of a sector.

You should be able to calculate the **arc length** of a **sector** using:

$$A.L. = \frac{angle}{360} \times \pi \times d$$

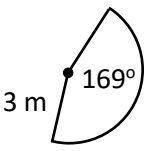
Find the arc length.



$A.L. = \frac{238}{360} \times \pi \times 12$

$= 24.92\ m$

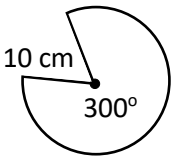
Find the arc length.



$A.L. = \frac{169}{360} \times \pi \times 6$

$= 8.85\ m$

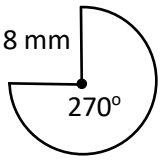
Find the arc length.



$A.L. = \frac{300}{360} \times \pi \times 20$

$= 52.36\ cm$

Find the arc length using $\pi = 3.14$. (**n-c**)



$A.L. = \frac{270}{360} \times 3.14 \times 16$

$= \frac{3}{4} \times 3.14 \times 16$

$= 12 \times 3.14$

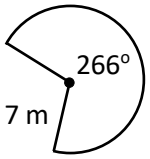
$= 37.68\ mm$

4.2 (b) I can calculate the area of a sector.

You should be able to calculate the **area** of a **sector** using:

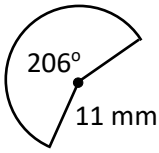
$$S.A. = \frac{angle}{360} \times \pi \times r^2$$

Find the sector area.



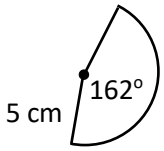
$$S.A. = \frac{266}{360} \times \pi \times 7^2$$
$$= 113.74\ m^2$$

Find the sector area.



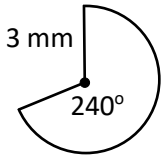
$$S.A. = \frac{206}{360} \times \pi \times 11^2$$
$$= 217.52\ mm^2$$

Find the sector area.



$$S.A. = \frac{162}{360} \times \pi \times 5^2$$
$$= 35.34\ cm^2$$

Find the area using $\pi = 3.14$. (**n-c**)



$$A.L. = \frac{240}{360} \times 3.14 \times 3^2$$
$$= \frac{2}{3} \times 3.14 \times 9$$
$$= 6 \times 3.14$$
$$= 18.84\ mm$$

4.2 (c) I can use the arc length to calculate the angle or radius.

In some questions you may be given the **arc length** or **sector area** in order to calculate an **angle** or **radius.**

Remember: Addition and Subtraction are **inverses**. Multiplication and division are likewise **inverses**.

A sector has arc length 32 cm and radius 6 cm.

Calculate the size of the angle.

$$A.L. = \frac{x}{360} \times \pi \times 12$$
$$32 = \frac{x}{360} \times \pi \times 12$$
$$32 \times 360 = x \times \pi \times 12$$
$$\frac{32 \times 360}{\pi \times 12} = x$$
$$x = 305.58^o$$

A sector has arc length 56 cm and angle 132°.

Calculate the length of the diameter.

$$A.L. = \frac{132}{360} \times \pi \times d$$
$$56 = \frac{132}{360} \times \pi \times d$$
$$56 \times 360 = 132 \times \pi \times d$$
$$\frac{56 \times 360}{132 \times \pi} = d$$
$$d = 48.61\ cm$$

A sector has area 72 cm^2 and radius 20 cm.

Calculate the size of the angle.

$$S.A. = \frac{x}{360} \times \pi \times 20^2$$
$$72 = \frac{x}{360} \times \pi \times 20^2$$
$$72 \times 360 = x \times \pi \times 20^2$$
$$\frac{72 \times 360}{\pi \times 20^2} = x$$
$$x = 20.63^o$$

A sector has arc length 16 cm and angle 42°.

Calculate the length of the radius.

$$A.L. = \frac{42}{360} \times \pi \times d$$
$$16 = \frac{42}{360} \times \pi \times d$$
$$16 \times 360 = 42 \times \pi \times d$$
$$\frac{16 \times 360}{42 \times \pi} = d$$
$$d = 43.65\ cm$$
$$r = 21.83\ cm$$

4.3 Gradient

4.3 (a) I can calculate the gradient of a straight line.

From National 4: You should know the following:

Gradient is a measure of how **steep** a **slope** is. If a gradient is **positive** it slopes **up**. If it is **negative** it slopes **down**. It can be calculated using:

$$\text{Gradient} = \frac{\text{vertical height}}{\text{horizontal distance}}$$

In National 5: To find the **gradient** between two points (x_1, y_1) and (x_2, y_2) you can use the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- If a line is **horizontal** it has a gradient of **zero**.
- If a line is **vertical** it has an **undefined** gradient.
- Two lines are **parallel** if they have the **same gradient**.

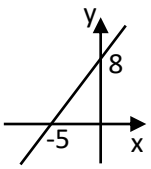
Find the gradient between the points (-2, -2) and (1, 7).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - -2}{1 - -2}$$
$$= \frac{9}{3}$$
$$= 3$$

Find the gradient between the points (-5, 1) and (1, -2).

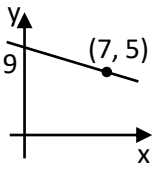
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - 1}{1 - -5}$$
$$= -\frac{3}{6}$$
$$= -\frac{1}{2}$$

Calculate the gradient of the straight line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - 8}{-5 - 0}$$
$$= \frac{8}{5}$$

Calculate the gradient of the straight line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 9}{7 - 0}$$
$$= -\frac{4}{7}$$

5. Linear Equations

5.1 Solving Equations

5.1 (a) I can solve equations involving brackets.

You are expected to be able to **solve equations involving brackets**.

$9m-2=2m+10$ $7m-2=10$ $7m=12$ $m=\frac{12}{7}$	$7x-2(x-6)=9$ $7x-2x+12=9$ $5x+12=9$ $5x=-3$ $x=-\frac{3}{5}$	$3(y+4)+1=y-7$ $3y+12+1=y-7$ $3y+13=y-7$ $2y+13=-7$ $2y=-20$ $y=-10$	$b^2+3=b(b-4)+2$ $b^2+3=b^2-4b+2$ $3=-4b+2$ $1=-4b$ $-\frac{1}{4}=b$

5.1 (b) I can solve equations involving fractions.

You are expected to be able to **solve equations involving fractions**.

$2f+3=\frac{f-4}{3}$ $6f+9=f-4$ $5f+9=-4$ $5f=-13$ $f=-\frac{13}{5}$	$\frac{5}{x}+7=10$ $5+7x=10x$ $5=3x$ $\frac{5}{3}=x$	$\frac{w}{2}+\frac{1}{6}=3$ $\frac{3w}{6}+\frac{1}{6}=3$ $3w+1=18$ $3w=17$ $w=\frac{17}{3}$	$\frac{m}{5}-\frac{1}{4}=m$ $\frac{4m}{20}-\frac{5}{20}=m$ $4m-5=20m$ $-5=16m$ $-\frac{5}{16}=m$

5.1 (c) I can solve inequations.

An **inequation** is like an **equation** but with a $>, <, \geq$ or $\leq$ instead of a $=$ sign.

When you multiply or divide both sides of an inequation by a negative the **inequality is reversed**.

$4x-5\leq x-20$ $3x-5\leq -20$ $3x\leq -15$ $x\leq -5$	$9-x<3(x+4)$ $9-x<3x+12$ $9<4x+12$ $-3<4x$ $-\frac{3}{4}<x$	$12+x>6+2(x-1)$ $12+x>6+2x-2$ $12+x>2x+4$ $12>x+4$ $8>x$	$3y-4\geq\frac{y+1}{2}$ $6y-8\geq y+1$ $5y-8\geq 1$ $5y\geq 9$ $y\geq\frac{9}{5}$

5.2 Straight Lines

5.2 (a) I can identify the equation of a straight line.

A **vertical line** passing through (3, 0) has equation $x = 3$ and **undefined gradient**. A **horizontal line** passing through (0, 7) has equation $y = 7$ and a **gradient of zero**.

More generally you should be able to identify the equation of a straight line**:**

1. Calculate the **gradient** of the line using any two points on the line.
2. **Substitute** into $y - b = m(x - a)$ where m is the gradient and (a, b) is a point on the line.
3. **Tidy up** and **rearrange** so that you have the line in the form $y = mx + c$.

Find the equation of the straight line with gradient -2 passing through (4, -7).

$y - b = m(x - a)$

$y - -7 = -2(x - 4)$

$y + 7 = -2x + 8$

$y = -2x + 1$

Find the equation of the straight line with gradient 3 passing through (-5, 9).

$y - b = m(x - a)$

$y - 9 = 3(x - -5)$

$y - 9 = 3x + 15$

$y = 3x + 24$

Find the equation of the straight line joining the two points (-2, 5) and (4, 17).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{17 - 5}{4 - -2}$

$= \frac{12}{6}$

$= 2$

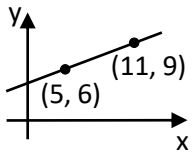
$y - b = m(x - a)$

$y - 5 = 2(x - -2)$

$y - 5 = 2x + 4$

$y = 2x + 9$

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - 6}{11 - 5}$

$= \frac{3}{6}$

$= \frac{1}{2}$

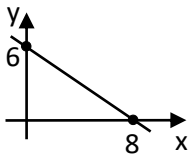
$y - b = m(x - a)$

$y - 6 = \frac{1}{2}(x - 5)$

$y - 6 = \frac{1}{2}x - \frac{5}{2}$

$y = \frac{1}{2}x + \frac{7}{2}$

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 6}{8 - 0}$

$= -\frac{6}{8}$

$= -\frac{3}{4}$

$y - b = m(x - a)$

$y - 6 = -\frac{3}{4}(x - 0)$

$y - 6 = -\frac{3}{4}x$

$y = -\frac{3}{4}x + 6$

5.2 (b) I can identify the gradient and y-intercept from an equation.

If you **rearrange** an equation into the form $y = mx + c$ this allows you to identify both the **gradient** (m) and the **y-intercept** (c).

Find the gradient of the line with equation $4x + 5y = 2$.

$5y = -4x + 2$

$y = -\frac{4}{5}x + \frac{2}{5}$

$m = -\frac{4}{5}$

Find the gradient and y-intercept of the line $2x + 3y - 7 = 0$.

$3y - 7 = -2x$

$3y = -2x + 7$

$y = -\frac{2}{3}x + \frac{7}{3}$

$m = -\frac{2}{3}, c = \frac{7}{3}$

Find the gradient of the line $5y + 6x = 10$.

$5y = -6x + 10$

$y = -\frac{6}{5}x + 2$

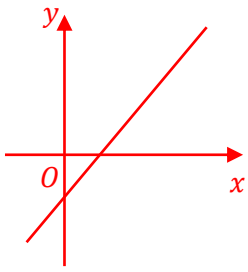
$m = -\frac{6}{5}$

The line crosses the y-axis at $(0, c)$. Find c.

$c = 2$

A straight line has equation $y = mx + c$.

Sketch a graph for this given $m > 0$ and $c < 0$.



5.2 (c) I can use substitution to identify coordinates.

If a point lies on a line, **substitute** the x-coordinate into the line's equation to find the y-coordinate and vice versa. On the **x-axis** $y = 0$ and on the **y-axis** $x = 0$.

Find the point of intersection of the line $y = -3x + 8$ and the y-axis.

On the y-axis $x = 0$.

$y = -3 \times 0 + 8$

$y = 8$

Coordinates $(0, 8)$.

Find the point of intersection of the line $y = 4x + 12$ and the x-axis.

On the x-axis $y = 0$.

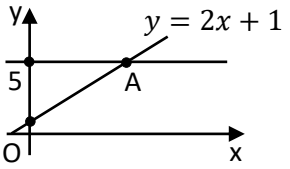
$0 = 4x + 12$

$-12 = 4x$

$x = -3$

Coordinates $(-3, 0)$.

Find the coordinate of A.



On the line $y = 5$.

$5 = 2x + 1$

$4 = 2x$

$2 = x$

Coordinates $(2, 5)$.

The point $(3m, m)$ lies on the line $y = 2x - 5$.

Find the value of m.

At $(3m, m)$ $x = 3m$, $y = m$.

$m = 2 \times 3m - 5$

$m = 6m - 5$

$-5m = -5$

$m = 1$

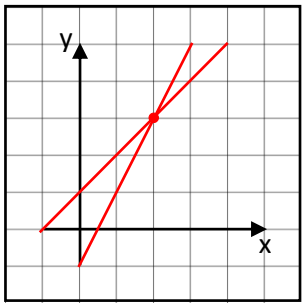
5.3 Simultaneous Eqns

5.3 (a) I can solve simultaneous equations graphically.

To solve a pair of equations you need to find values which satisfy **both** equations **simultaneously**.

You solve **simultaneous equations graphically** by drawing the graph of each equation on the same diagram and finding their **point of intersection**.

Solve $y - 2x = -1$ and $y - 1 = x$ graphically.



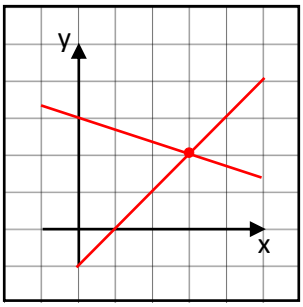
$y = 2x - 1$

$y = x + 1$

Intersection $(2, 3)$.

So $x = 2$ & $y = 3$.

Solve $y - x = -1$ and $3y = -x + 9$ graphically.



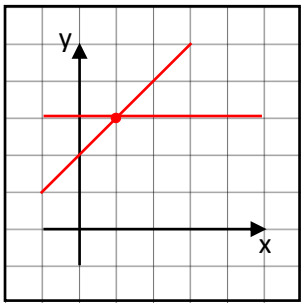
$y = x - 1$

$y = -1/3x + 3$

Intersection $(3, 2)$.

So $x = 3$ & $y = 2$.

Solve $y - x = 2$ and $y = 3$ graphically.



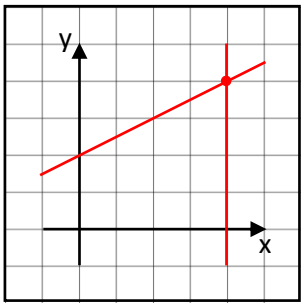
$y = x + 2$

$y = 3$

Intersection $(1, 3)$.

So $x = 1$ & $y = 3$.

Solve $2y = x + 4$ and $x = 4$ graphically.



$y = 1/2x + 2$

$x = 4$

Intersection $(4, 4)$.

So $x = 4$ & $y = 4$.

5.3 (b) I can solve simultaneous equations algebraically.

To solve **simultaneous equations algebraically**:

1. **Size** the equations so that the **coefficients are the same.**
2. **Add or subtract** the equations to **eliminate** this term.
3. **Solve** the resulting equation to find the remaining term.
4. **Substitute** this value into either equation to find the other term.

Solve $3x + 2y = 8$ and $2x - y = 3$.

$3x + 2y = 8 \xrightarrow{\times 2} 6x + 4y = 16$

$2x - y = 3 \xrightarrow{\times 3} 6x - 3y = 9$

$7y = 7$

$y = 1$

$y = 1 \Rightarrow 2x - 1 = 3$

$2x = 4$

$x = 2$

Find the solution of $a + b = 5$ and $5a - 3b = 3$.

$a + b = 5 \xrightarrow{\times 5} 5a + 5b = 25$

$5a - 3b = 3 \rightarrow 5a - 3b = 3$

$8b = 22$

$b = 2.75$

$b = 2.75 \Rightarrow a + 2.75 = 5$

$a = 2.25$

Find the point of intersection of $2x + 3y = 8$ and $3x + 2y = 7$.

$2x + 3y = 8 \xrightarrow{\times 3} 6x + 9y = 24$

$3x + 2y = 7 \xrightarrow{\times 2} 6x + 4y = 14$

$5y = 10$

$y = 2$

$y = 2 \Rightarrow 2x + 6 = 8$

$2x = 2$

$x = 1$

$Coordinates\ (1, 2).$

5.3 (c) I can construct and solve simultaneous equations in context.

You should also be able to **construct** and **solve simultaneous equations** in **real life settings**.

To get into a carnival; 2 adults and 2 children pay £54 while 1 adult and 4 children pay £51.

Construct two equations to represent these.

$2a + 2c = 54$

$a + 4c = 51$

Find the cost of an adult's ticket and the cost of a child's ticket.

$2a + 2c = 54 \rightarrow 2a + 2c = 54$

$a + 4c = 51 \xrightarrow{\times 2} 2a + 8c = 102$

$6c = 48$

$c = 8$

$c = 8 \Rightarrow a + 32 = 51$

$a = 19$

An adult's ticket costs £19 and a child's ticket costs £8.

Aidan buys 2 teas and 3 coffees for £6.60 and Ruth buys 4 teas and 1 coffee for £7.20.

Construct two equations to represent these.

$2t + 3c = 6.60$

$4t + c = 7.20$

Find the price of a tea and the price of a coffee.

$2t + 3c = 6.6 \xrightarrow{\times 2} 4t + 6c = 13.2$

$4t + c = 7.2 \rightarrow 4t + c = 7.2$

$5c = 6$

$c = 1.2$

$c = 1.2 \Rightarrow 2t + 3.6 = 6.6$

$2t = 3$

$t = 1.5$

An tea costs £1.50 and a coffee costs £1.20.

Side by side a wardrobe and 3 desks measure 330 cm meanwhile 2 wardrobes and 2 desks measure 380 cm.

Construct equations for both.

$w + 3d = 330$

$2w + 2d = 380$

Find the measurements for both a wardrobe and a desk.

$w + 3d = 330 \xrightarrow{\times 2} 2w + 6d = 660$

$2w + 2d = 380 \rightarrow 2w + 2d = 380$

$4d = 280$

$d = 70$

$d = 70 \Rightarrow w + 210 = 330$

$w = 120$

An wardrobe measures 120 cm and a desk measures 70 cm.

5.4 Rearranging

5.4 (a) I can rearrange using the four basic operations.

A **formula** is an equation which shows the relationship between different quantities. You can **rearrange** a formula involving **addition**, **subtraction**, **multiplication** and **division**.

Note: Another way of saying rearrange is '**change the subject of the formula**'.

Rearrange for x:	Rearrange for a:	Rearrange for B:	Rearrange for C:	Rearrange for x:
$y = 5x - 2.$	$\frac{2a}{5} + b^2 = 7.$	$P = 3(A + B).$	$F = \frac{2C - y}{7}.$	$R = \frac{kx}{d} - e.$
$y + 2 = 5x$	$\frac{2a}{5} = 7 - b^2$	$\frac{P}{3} = A + B$	$7F = 2C - y$	$R + e = \frac{kx}{d}$
$\frac{y + 2}{5} = x$	$2a = 5(7 - b^2)$	$\frac{P}{3} - A = B$	$7F + y = 2C$	$d(R + e) = kx$
	$a = \frac{5(7 - b^2)}{2}$		$\frac{7F + y}{2} = C$	$\frac{d(R + e)}{k} = x$

5.4 (b) I can rearrange using squares and square roots.

You can also rearrange a formula involving **powers and roots**.

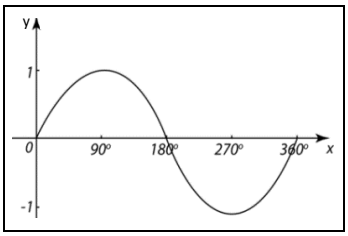
Rearrange for r:	Rearrange for b:	Rearrange for r:	Rearrange for t:	Rearrange for x:
$A = \frac{\pi r^2}{3}.$	$L = 3a + \sqrt{b}.$	$A = \sqrt{2rt}.$	$Z = \sqrt{4at - q}.$	$a = \frac{5x^2 + b}{3}$
$3A = \pi r^2$	$L - 3a = \sqrt{b}$	$A^2 = 2rt$	$Z^2 = 4at - q$	$3a = 5x^2 + b$
$\frac{3A}{\pi} = r^2$	$(L - 3a)^2 = b$	$\frac{A^2}{2t} = r$	$Z^2 + q = 4at$	$3a - b = 5x^2$
$\sqrt{\frac{3A}{\pi}} = r$			$\frac{Z^2 + q}{4a} = t$	$\frac{3a - b}{5} = x^2$
				$\sqrt{\frac{3a - b}{5}} = x$

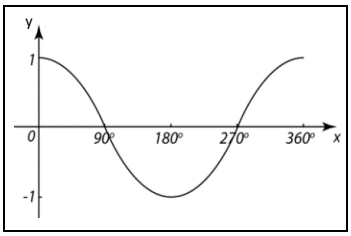
6. Trigonometric Graphs

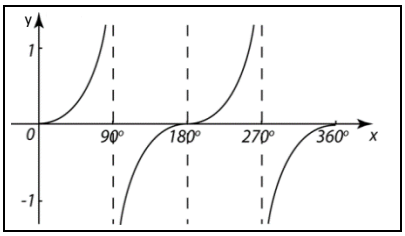
6.1 Trig Graphs

6.1 (a) I can work with the sine, cosine and tangent graphs.

You should know the graphs of the functions $y = \sin x^o$, $y = \cos x^o$, and $y = \tan x^o$.







You should know the following terms:

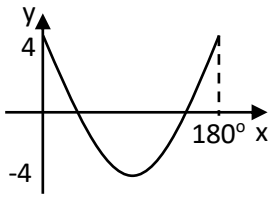
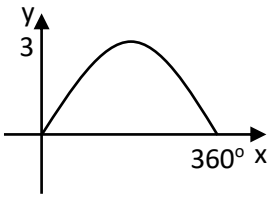
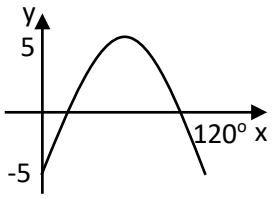
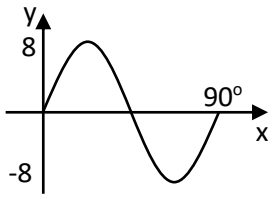
- The **amplitude** is the distance from the midpoint to the highest/lowest point of the graph.
- The **period** of a graph is the number of degrees it takes the graph to do one complete cycle.
- The **frequency** of a **sine** or **cosine** graph is how many times it repeats in 360°.
- The **frequency** of a **tangent** graph is how many times it repeats in 180°.

State the period of $y = \sin 3x^o$. $Period = 360^o \div 3$ $= 120^o$	State the period of $y = \cos 4x^o$. $Period = 360^o \div 4$ $= 90^o$	Write in increasing order. $\sin 120^o$ $\sin 270^o$ $\sin 305^o$ $\sin 270^o\ (as\ -1)$ $\sin 305^o\ (as\ negative)$ $\sin 120^o\ (as\ positive)$	Write in increasing order. $\cos 90^o$ $\cos 300^o$ $\cos 165^o$ $\cos 165^o\ (as\ negative)$ $\cos 90^o\ (as\ zero)$ $\cos 300^o\ (as\ positive)$

6.1 (b) I can identify graphs using amplitudes and frequencies.

You should be able to identify a **graph** from its **amplitude** and **frequency.**

$y = a \sin bx^o$	The **height** of the graph will vary between $-a$ and a. The **graph** will fit into 360° b times

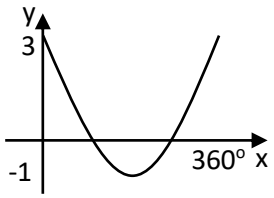
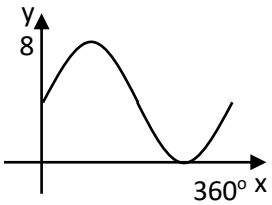
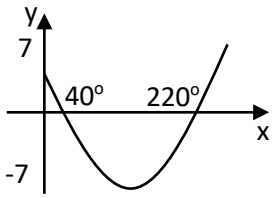
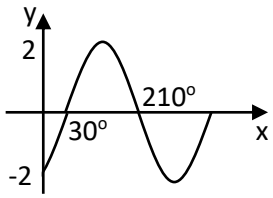
Write down the equation of $y = a \cos bx^o$.  $y = 4 \cos 2x^o$	Write down the equation of $y = a \sin bx^o$.  $y = 3 \sin 0.5x^o$	Write down the equation of $y = a \cos bx^o$.  $y = -5 \cos 3x^o$	Write down the equation of $y = a \sin bx^o$.  $y = 8 \sin 4x^o$

6.1 (c) I can identify graphs using phase angles and vertical translations.

You should also be able to identify a **graph** from its **phase angle** and **vertical translation**.

- The **phase angle** is the amount a graph has been **shifted to the right**.
- A **vertical translation** is the amount a graph has been **shifted up or down**.

$y = \cos(x - a)^o + b$	The **graph** will be pulled a to the **right**. The **graph** will be pulled b up.

Write down the equation of $y = a\cos x^o + b$.  $y = 2\cos x^o + 1$	Write down the equation of $y = a\sin x^o + b$.  $y = 4\sin x^o + 4$	Write down the equation of $y = a\cos(x - b)^o$.  $y = 7\cos(x + 50)^o$	Write down the equation of $y = a\sin(x - b)^o$.  $y = 2\sin(x - 30)^o$

6.2 Trig Equations

6.2 (a) I can solve basic trigonometric equations.

You can recognise a **trigonometric equation** because it will have the phrase: 'solve … for $0 \leq x < 360$'.

1. Take the **inverse** using your calculator.
2. Use the **CAST** diagram to identify the **quadrants**.
3. Use these to give you your **two solutions.**

$180 - x$	S	A x
$180 + x$	T	C $360 - x$

Solve $\cos x = 0.456$ for $0 \leq x < 360$. $x = \cos^{-1}(0.456)$ $= 62.87^o$ $\cos x + ve$ so $Q1$ & $Q4$ $x = 62.87^o,$ $(360 - 62.87)^o$ $= 62.87^o, 297.13^o$	Solve $\tan x = -3$ for $0 \leq x < 360$. $x = \tan^{-1}(3)$ $= 71.57^o$ $\tan x - ve$ so $Q2$ & $Q4$ $x = (180 - 71.57)^o,$ $(360 - 71.57)^o$ $= 108.43^o, 288.43^o$	Solve $\sin x = 0.98$ for $0 \leq x < 360$. $x = \sin^{-1}(0.98)$ $= 78.52^o$ $\sin x + ve$ so $Q1$ & $Q2$ $x = 78.52^o,$ $(180 - 78.52)^o$ $= 78.52^o, 101.48^o$	Solve $\sin x = -0.871$ for $0 \leq x < 360$. $x = \sin^{-1}(0.871)$ $= 60.58^o$ $\sin x - ve$ so $Q3$ & $Q4$ $x = (180 + 60.58)^o,$ $(360 - 60.58)^o$ $= 240.58^o, 299.42^o$

6.2 (b) I can rearrange and solve trigonometric equations.

Sometimes you will need to **rearrange** a **trigonometric equation** before solving it.

Solve $\cos x + 2 = 1.4$ for $0 \leq x < 360$. $\cos x = -0.6$ $x = \cos^{-1}(0.6)$ $= 53.13^o$ $\cos x - ve\ so\ Q2\ \&\ Q3$ $x = (180 - 53.13)^o,$ $(180 + 53.13)^o$ $= 126.87^o, 233.13^o$	Solve $5\sin x - 2 = 0$ for $0 \leq x < 360$. $5\sin x = 2$ $\sin x = 0.4$ $x = \sin^{-1}(0.4)$ $= 23.58^o$ $\sin x + ve\ so\ Q1\ \&\ Q2$ $x = 23.58^o,$ $(180 - 23.58)^o$ $= 23.58^o, 156.42^o$	Solve $4\tan x - 1 = -9$ for $0 \leq x < 360$. $4\tan x = -8$ $\tan x = -2$ $x = \tan^{-1}(2)$ $= 63.43^o$ $\tan x - ve\ so\ Q2\ \&\ Q4$ $x = (180 - 63.43)^o,$ $(360 + 63.43)^o$ $= 116.57^o, 296.57^o$	Solve $2\cos x - 1 = 0$ for $0 \leq x < 360$. $2\cos x = 1$ $\cos x = 0.5$ $x = \cos^{-1}(0.5)$ $= 60^o$ $\cos x + ve\ so\ Q1\ \&\ Q4$ $x = 60^o,$ $(360 - 60)^o$ $= 60^o, 300^o$

6.2 (c) I can use trigonometric identities to simplify and prove statements.

There are two important **trigonometric identities** which you are expected to know and to be able to work with.

$$\tan x^o = \frac{\sin x^o}{\cos x^o} \qquad \sin^2 x^o + \cos^2 x^o = 1$$

The second identity can be **rearranged** to either:

- $\sin^2 x^o = 1 - \cos^2 x^o$ or
- $\cos^2 x^o = 1 - \sin^2 x^o$.

Simplify: $\tan x \cos x.$ $= \frac{\sin x}{\cancel{\cos x}}\cancel{\cos x}$ $= \sin x$	Simplify: $\cos^3 x + \sin^2 x \cos x.$ $= \cos x\,(\cos^2 x + \sin^2 x)$ $= \cos x$	Simplify: $\sin^3 x + \sin x \cos^2 x.$ $= \sin x\,(\sin^2 x + \cos^2 x)$ $= \sin x$	Simplify: $\frac{2\sin^2 x}{1 - \sin^2 x}.$ $= \frac{2\sin^2 x}{\cos^2 x}$ $= 2\tan^2 x$

7. Statistical Skills

7.1 Standard Deviation

7.1 (a) I can calculate the mean and standard deviation of a data set.

The **standard deviation** of a set of numbers is a measure of how spread out they are from the **mean**. It is calculated using the following formula given in the **exam formulae sheet**:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

where $\bar{x}$ is the **mean**, n is the **number of numbers** and $\sum(x-\bar{x})^2$ is the **sum of the differences** between the set of numbers and the mean.

Calculate the mean and standard deviation of 2, 3, 9, 6, 5.

$\bar{x} = \frac{2+\cdots+5}{5} = \frac{25}{5} = 5$

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	−3	9
3	−2	4
5	0	0
6	1	1
9	4	16
25	0	30

$s = \sqrt{\frac{30}{5-1}} = 2.74$

Calculate the mean and standard deviation 21, 17, 14, 24, 19.

$\bar{x} = \frac{21+\cdots+19}{5} = \frac{95}{5} = 19$

x	$x-\bar{x}$	$(x-\bar{x})^2$
21	2	4
17	−2	4
14	−5	25
24	5	25
19	0	0
95	0	58

$s = \sqrt{\frac{58}{5-1}} = 3.81$

Calculate the mean and standard deviation 4, 10, 8, 7, 3, 4.

$\bar{x} = \frac{4+\cdots+4}{6} = \frac{36}{6} = 6$

x	$x-\bar{x}$	$(x-\bar{x})^2$
4	−2	4
10	4	16
8	2	4
7	1	1
3	−3	9
4	−2	4
36	0	38

$s = \sqrt{\frac{38}{6-1}} = 2.76$

When comparing the **mean** and **standard deviation** of two data sets:

- Use the expression '**on average**' instead of **mean.**
- A **larger standard deviation** means '**more varied**' or '**less consistent**'.
- **Never** use both **varied** and **consistent** in the same example.

In March the mean temperature in Inverness is 7°C and the standard deviation is 2.5. In London meanwhile they are 11°C and 3 respectively.

Compare the cities' temperatures.

On average the temperature in London is higher.

The temperatures in London are more varied.

The mean number of goals scored by Dundee was 3.2 per game and for Falkirk 2.8. The standard deviation for Dundee was 0.9 and for Falkirk 1.3.

Compare the number of goals scored by the clubs.

On average the number of goals scored a game by Dundee is higher.

The number of goals scored by Falkirk was more varied.

In their Unit One test a class had a mean score of 58 and a standard deviation of 11.2. In their Unit Two test these were 55 and 7.6 respectively.

Compare the class' performance in the tests.

On average the test scores were better in the Unit One test.

The number of marks was more varied in the Unit One test.

The mean number of hours that pupils had spent studying for a test was 15 and the standard deviation was 3.4.

The next year they were 15 and 4.2. Comment on these.

On average the number of hours spent revising was the same.

The number of hours spent revising by the pupils was more varied the second year.

7.2 Quartiles

7.2 (a) I can find the quartiles of a data set.

You are expected to work with **quartiles** -these divide an **ordered list** into **four equally-sized parts.**

5, 2, 1, 7, 4, 1, 1. $1\ (1)\ 1\ (2)\ 4\ (5)\ 7$ $Q_1 \quad Q_2 \quad Q_3$	42, 50, 63, 55, 49. $42 \mid 49\ (50)\ 55 \mid 63$ $45.5 \quad Q_2 \quad 59$ $Q_1 \quad\quad Q_3$	22, 27, 23, 29, 35, 18. $18\ (22)\ 23 \mid 27\ (29)\ 35$ $Q_1 \quad 25 \quad Q_3$ Q_2	4, 8, 3, 4, 5, 8, 1, 9. $1\ \ 3 \mid 4\ \ 4 \mid 5\ \ 8 \mid 8\ \ 9$ $3.5 \quad 4.5 \quad 8.5$ $Q_1 \quad Q_2 \quad Q_3$

7.2 (b) I can find the interquartile and semi-interquartile ranges.

You should also be able to calculate **interquartile** and **semi-interquartile ranges**:

- The **Interquartile range** is the difference between the upper and lower quartiles: $IQR = Q3 - Q1$
- The **Semi-Interquartile range** is half of this: $SIQR = (Q3 - Q1) \div 2$

You can compare the **median** and **semi-interquartile ranges** of two data sets in the same way that you would the **mean** and **standard deviation** of two data sets.

- Use the expression '**on average**' instead of **median.**
- A **larger semi-interquartile range** means '**more varied**' or '**less consistent**'.

Find the IQR of: 4, 3, 8, 7, 2, 9. $2\ (3)\ 4 \mid 7\ (8)\ 9$ $Q_1 \quad 5.5 \quad Q_3$ Q_2 $IQR = 8 - 3$ $= 5$	Find the SIQR of: 9, 7, 6, 9, 8, 4, 5. $4\ (5)\ 6\ (7)\ 8\ (9)\ 9$ $Q_1 \quad Q_2 \quad Q_3$ $SIQR = (9 - 5) \div 2$ $= 2$	Find the SIQR of: 1, 4, 2, 4, 7, 9, 3, 8. $1\ \ 2 \mid 3\ \ 4 \mid 4\ \ 7 \mid 8\ \ 9$ $2.5 \quad 4 \quad 7.5$ $Q_1 \quad Q_2 \quad Q_3$ $SIQR = (7.5 - 2.5) \div 2$ $= 2.5$	If $Q_1 = 12$, $Q_2 = 14$ and $Q_3 = 18$, find the IQR and SIQR. $IQR = 18 - 12$ $= 6$ $SIQR = 6 \div 2$ $= 3$

7.3 Scatter Graphs

7.3 (a) I can find and use the equation of a scatter graph's line of best fit.

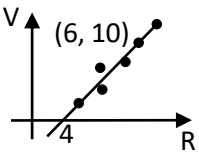
From National 4: You should know the following:

By looking at a **scatter graph**, you can see whether there is any connection (**correlation**) between the two sets of data. If there is a **line of best fit** it can be plotted and then used.

In National 5: You are expected to be able to calculate the **equation of a line of best fit**:

1. Calculate the **gradient** of the line using any two points on the line.
2. **Substitute** into $y - b = m(x - a)$ where m is the gradient and (a, b) is a point on the line.
3. **Tidy up** and **rearrange** so that you have the line in the form $y = mx + c$.
4. **If the letters on the graph are not y and x change the equation for these new letters.**

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - 0}{6 - 4}$

$= \frac{10}{2}$

$= 5$

$y - b = m(x - a)$

$y - 0 = 5(x - 4)$

$y = 5x - 20$

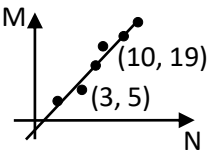
$V = 5R - 20$

Find V if R is 18.

$S = 5 \times 18 - 20$

$= 70$

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{19 - 5}{10 - 3}$

$= \frac{14}{7}$

$= 2$

$y - b = m(x - a)$

$y - 5 = 2(x - 3)$

$y - 5 = 2x - 6$

$y = 2x - 1$

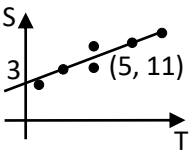
$M = 2N - 1$

Find M if N is 15.

$M = 2 \times 15 - 1$

$= 29$

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{11 - 3}{5 - 0}$

$= \frac{8}{5}$

$y - b = m(x - a)$

$y - 3 = \frac{8}{5}(x - 0)$

$y - 3 = \frac{8}{5}x$

$y = \frac{8}{5}x + 3$

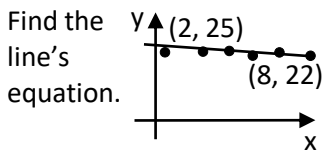
$S = \frac{8}{5}T + 3$

Find S if T is 10.

$S = \frac{8}{5} \times 10 + 3$

$= 19$

Find the line's equation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{22 - 25}{8 - 2}$

$= \frac{-3}{6}$

$= -\frac{1}{2}$

$y - b = m(x - a)$

$y - 25 = -\frac{1}{2}(x - 2)$

$y - 25 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 26$

Find P if R is 12.

$P = -\frac{1}{2} \times 12 + 26$

$= 20$

8. Quadratics

8.1 Completed Square

8.1 (a) I can write an expression in the completed square form.

A **quadratic expression** $x^2 + bx + c$ can be written in in the **completed square form** $(x + p)^2 + q$.

To do this the following phrase can be useful: **'half it, square it and take it away'**.

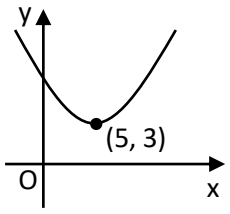
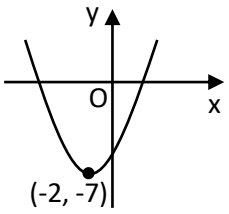
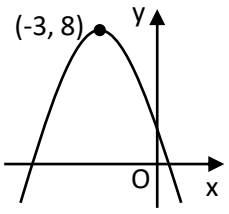
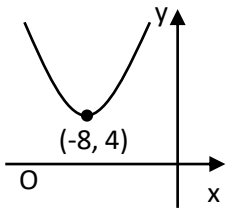
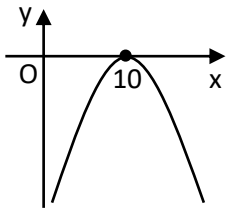
Write $x^2 - 4x$ in the form $(x + a)^2 + b$.	Write $x^2 + 2x - 9$ in the form $(x + a)^2 + b$.	Write $x^2 - 6x + 20$ in the form $(x + a)^2 + b$.	Write $x^2 + 5x + 1$ in the form $(x + a)^2 + b$.
$= (x - 2)^2 - 4$	$= (x + 1)^2 - 1 - 9$	$= (x - 3)^2 - 9 + 20$	$= (x + 2.5)^2 - 6.25 + 1$
	$= (x + 1)^2 - 10$	$= (x - 3)^2 + 11$	$= (x + 2.5)^2 - 5.25$

8.1 (b) I can identify the equation of a parabola in completed square form.

A **parabola's** equation can be identified in the **completed square form** from its **turning point**. If a parabola has turning point (a, b) then it has equation:

$$y = (x - a)^2 + b$$

Note: If a **parabola** has a ∩ shape instead of a ∪ shape then it must have the form $y = -(x - c)^2 + d$.

				
$y = (x - 5)^2 + 3$	$y = (x + 2)^2 - 7$	$y = -(x + 3)^2 + 8$	$y = (x + 8)^2 + 4$	$y = -(x - 10)^2$

8.1 (c) I can identify the symmetry and turning point of a parabola.

Once in **completed square form** you can identify a parabola's **symmetry**, **turning point** and **nature**.

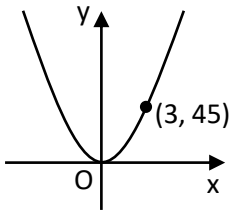
Equation	Turning Point	Nature	Line of Symmetry	Shape of Graph
$y = (x - a)^2 + b$	(a, b)	Minimum	$x = a$	∪
$y = -(x - c)^2 + d$	(c, d)	Maximum	$x = c$	∩

$y = (x - 8)^2 + 1.$	$y = -(x + 4)^2 + 9.$	$y = (x - 3)^2 - 7.$	$y = (x + 5)^2 - 6.$	$y = 7 - (x - 2)^2.$
$T.P.\ (8, 1).$	$T.P.\ (-4, 9).$	$T.P.\ (3, -7).$	$T.P.\ (-5, -6).$	$T.P.\ (2, 7).$
$Minimum$	$Maximum$	$Minimum$	$Minimum$	$Maximum$
$Symmetry\ x = 8.$	$Symmetry$ $x = -4.$	$Symmetry\ x = 3.$	$Symmetry$ $x = -5.$	$Symmetry\ x = 2.$

8.1 (d) I can work with quadratics and substitution.

If you are given a **point** on a quadratic and asked to find its equation **substitution** is required.

$y = kx^2$ find k.



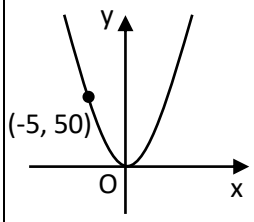
$y = kx^2$

$45 = k \times 3^2$

$45 = k \times 9$

$5 = k$

$y = mx^2$ find m.



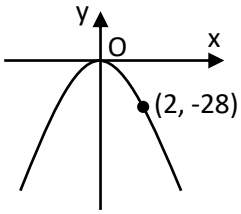
$y = mx^2$

$50 = m \times (-5)^2$

$50 = m \times 25$

$2 = m$

$y = ax^2$ find a.



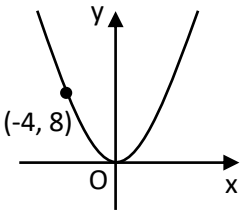
$y = ax^2$

$-28 = a \times 2^2$

$-28 = a \times 4$

$-7 = a$

$y = bx^2$ find b.



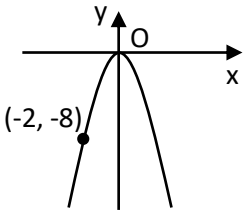
$y = bx^2$

$8 = b \times (-4)^2$

$8 = b \times 16$

$0.5 = b$

$y = px^2$ find p.



$y = px^2$

$-8 = p \times (-2)^2$

$-8 = p \times 4$

$-2 = p$

Substitution can also be used to solve problems in slightly different contexts.

A parabola $y = (x - a)^2 + b$ has a line of symmetry $x = 2$ and passes through (5, 14). Find a and b.

$a = 2$

$At\ (5, 14)\ x = 5, y = 14.$

$14 = (5 - 2)^2 + b$

$14 = 9 + b$

$5 = b$

A parabola $y = (x - a)^2 + b$ has a line of symmetry $x = 4$ and passes through (6, 30). Find a and b.

$a = 4$

$At\ (6, 30)\ x = 6, y = 30.$

$30 = (6 - 4)^2 + b$

$30 = 4 + b$

$26 = b$

8.1 (e) I can sketch a quadratic graph in completed square form.

You can be asked to **sketch a parabola** in **completed square form**, to do this:

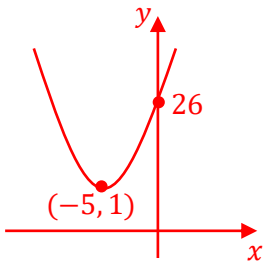
1. Plot the **turning point** (a, b).
2. Find and plot the **y-intercept** (using $x = 0$).
3. Draw a parabola passing through these points.

Sketch the graph of $y = (x + 5)^2 + 1$.

$On\ the\ y - axis\ x = 0.$

$y = (0 + 5)^2 + 1$

$= 26$

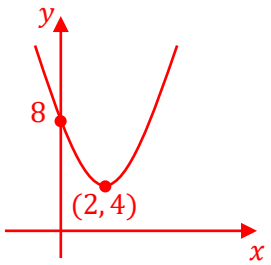


Sketch the graph of $y = (x - 2)^2 + 4$.

$On\ the\ y - axis\ x = 0.$

$y = (0 - 2)^2 + 4$

$= 8$

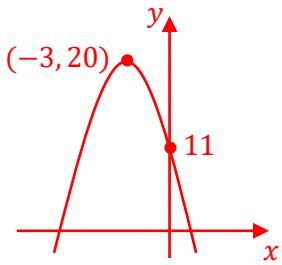


Sketch the graph of $y = -(x + 3)^2 + 20$.

$On\ the\ y - axis\ x = 0.$

$y = -(0 + 3)^2 + 20$

$= 11$

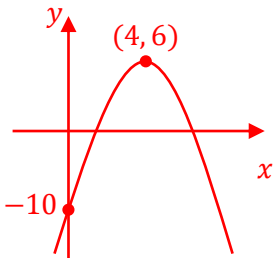


Sketch the graph of $y = 6 - (x - 4)^2$.

$On\ the\ y - axis\ x = 0.$

$y = 6 - (0 - 4)^2$

$= -10$



8.2 Factorised Form

8.2 (a) I can sketch a quadratic graph in factorised form.

Factorisation can allow you to write a **quadratic** in the form $y = (x - a)(x - b)$ where $(a, 0)$ and $(b, 0)$ are the **roots** of the equation and the **line of symmetry** is found halfway between these. To sketch such a graph:

1. Find and plot the **roots** $(a, 0)$ and $(b, 0)$.
2. Find the **y-intercept** (using $x = 0$).
3. Find the x-coordinate of the **turning point** – halfway between a and b.
4. Find the corresponding y-coordinate using **substitution**.
5. Draw a parabola passing through these points.

Sketch the graph of $y = (x - 3)(x + 7)$.

$On\ the\ y - axis\ x = 0.$

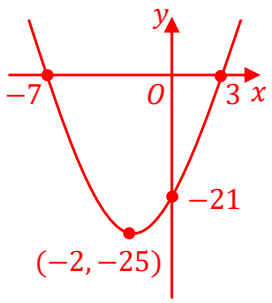
$y = (0 - 3)(0 + 7)$

$= -21$

$Symmetry\ x = -2.$

$y = (-2 - 3)(-2 + 7)$

$= -25$



Sketch the graph of $y = (x - 8)(x + 2)$.

$On\ the\ y - axis\ x = 0.$

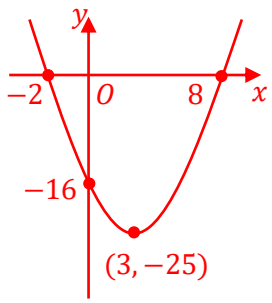
$y = (0 - 8)(0 + 2)$

$= -16$

$Symmetry\ x = 3.$

$y = (3 - 8)(3 + 2)$

$= -25$



If a parabola has roots at (-2, 0) and (8, 0) find its axis of symmetry.

$x = 3$

If a parabola has roots at (-9, 0) and (5, 0) find its axis of symmetry.

$x = -2$

If a parabola has roots at (-12, 0) and (-2, 0) find its axis of symmetry.

$x = -7$

A parabola has axis of symmetry at x = 9 and one root at (5, 0). Find the other root.

$Coordinates\ (13, 0).$

A parabola has axis of symmetry at x = 2 and one root at (-3, 0). Find the other root.

$Coordinates\ (7, 0).$

A parabola has axis of symmetry at x = -4 and one root at (8, 0). Find the other root.

$Coordinates\ (-16, 0).$

8.2 (b) I can find the roots of a quadratic equation using factorisation.

To solve a **quadratic equation** of the form $ax^2 + bx + c = 0$:

1. **Factorise** the left-hand side of the equation.
2. Set each factor equal to **zero**.
3. **Solve** both of the resulting equations.

$y^2 - 15y + 56 = 0.$

$(y - 7)(y - 8) = 0$

$y - 7 = 0 \qquad y - 8 = 0$

$y = 7 \qquad y = 8$

$x^2 + 4x - 21 = 0.$

$(x + 7)(x - 3) = 0$

$x + 7 = 0 \qquad x - 3 = 0$

$x = -7 \qquad x = 3$

$2m^2 - 7m + 3 = 0.$

$(2m - 1)(m - 3) = 0$

$2m - 1 = 0 \qquad m - 3 = 0$

$2m = 1 \qquad m = 3$

$m = 1/2$

$2x^2 + 6x + 4 = 0.$

$2(x^2 + 3x + 2) = 0$

$2(x + 2)(x + 1) = 0$

$x + 2 = 0 \qquad x + 1 = 0$

$x = -2 \qquad x = -1$

8.2 (c) I can rearrange, factorise and solve quadratic equations.

Sometimes you need to **rearrange** an equation to get it in the form $ax^2 + bx + c = 0$ first.

$x^2 - 5x = 24.$

$x^2 - 5x - 24 = 0$

$(x + 3)(x - 8) = 0$

$x + 3 = 0 \qquad x - 8 = 0$

$x = -3 \qquad x = 8$

$y^2 = 6y + 16.$

$y^2 - 6y = 16$

$y^2 - 6y - 16 = 0$

$(y + 2)(y - 8) = 0$

$y + 2 = 0 \qquad y - 8 = 0$

$y = -2 \qquad y = 8$

$x(x - 3) = 4.$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x + 1 = 0 \qquad x - 4 = 0$

$x = -1 \qquad x = 4$

If $f(x) = x^2 + 5x$ and $g(x) = x + 12$ solve the equation $f(x) = g(x)$.

$x^2 + 5x = x + 12$

$x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x + 6 = 0 \qquad x - 2 = 0$

$x = -6 \qquad x = 2$

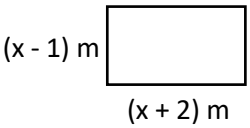
8.2 (d) I can solve quadratic equations in context.

Sometimes you may be given a **quadratic equation** in the context of a **graph** or concerning **shape properties**.

A rectangular lawn has an area of 40 square metres.

(a) Show $x^2 + x - 42 = 0$.

(b) Find the length x.



$Area = (x + 2)(x - 1)$

×	x	$+2$
x	x^2	$+2x$
-1	$-x$	-2

$Area = x^2 + x - 2$

$40 = x^2 + x - 2$

$0 = x^2 + x - 42$

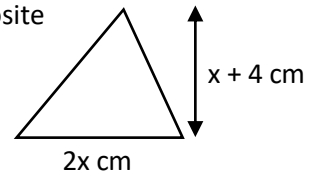
$(x + 7)(x - 6) = 0$

$x + 7 = 0 \qquad x - 6 = 0$

$x = -7 \qquad x = 6$

$x = 6\ m\ as\ -7\ is\ negative$

The area of the triangle opposite is 60 square centimetres



Calculate the value of x.

$Area = \frac{1}{2} \times 2x(x + 4)$

$60 = x(x + 4)$

$60 = x^2 + 4x$

$0 = x^2 + 4x - 60$

$(x + 10)(x - 6) = 0$

$x + 10 = 0 \qquad x - 6 = 0$

$x = -10 \qquad x = 6$

$x = 6\ cm\ as\ -10\ is\ negative$

8.3 Quadratic Formula

8.3 (a) I can use the quadratic formula to solve problems.

Some **quadratic equations** do not factorise, for these the **quadratic formula** can be used.

The **roots** of $ax^2 + bx + c = 0$ are given by the **quadratic formula**, this is found in the **exam formulae sheet:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the roots of $x^2 - 5x + 3 = 0$ to 1 decimal place. $a = 1, b = -5, c = 3$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1}$ $= \frac{5 \pm \sqrt{13}}{2}$ $= 4.303,\ 0.697$ $= 4.3,\ 0.7$	Find the roots of $2x^2 + 6x - 7 = 0$ to 2 significant figures. $a = 2, b = 6, c = -7$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times (-7)}}{2 \times 2}$ $= \frac{-6 \pm \sqrt{92}}{4}$ $= 0.898,\ -3.897$ $= 0.90,\ -3.9$	Find the roots of $3x^2 + 8x - 1 = 0$ to 3 decimal places. $a = 3, b = 8, c = -1$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-8 \pm \sqrt{8^2 - 4 \times 3 \times (-1)}}{2 \times 3}$ $= \frac{-8 \pm \sqrt{76}}{6}$ $= 0.1196,\ -2.7863$ $= 0.120,\ -2.786$

8.3 (b) I can use the discriminant to determine the nature of roots.

To determine the **nature of the roots** of the equation $ax^2 + bx + c = 0$ you can calculate the **discriminant**:

$$\boxed{b^2 - 4ac}$$

- If $b^2 - 4ac > 0$ then the quadratic equation has **two real distinct roots**.
- If $b^2 - 4ac = 0$ then the quadratic equation has **equal real roots**.
- If $b^2 - 4ac < 0$ then the quadratic equation has **no real roots**.

Determine the nature of the roots of the equation $3x^2 + 2x + 1 = 0$. $a = 3, b = 2, c = 1$ $b^2 - 4ac = 2^2 - 4 \times 3 \times 1$ $= -8$ *So no real roots*	Determine the nature of the roots of the equation $9x^2 + 3x - 4 = 0$. $a = 9, b = 3, c = -4$ $b^2 - 4ac = 3^2 - 4 \times 9 \times (-4)$ $= 153$ *So two real distinct roots*	Determine the nature of the roots of the equation $x^2 - 7x + 1 = 0$. $a = 1, b = -7, c = 1$ $b^2 - 4ac = (-7)^2 - 4 \times 1 \times 1$ $= 45$ *So two real distinct roots*

9. Surds & Indices

9.1 Surds

9.1 (a) I can simplify basic surds.

A **surd** is a root which does **not** have an exact answer. To **simplify** a surd, express it as a product of two numbers, one of which is a **square**.

Note: The **square** numbers to look out for are: 4, 9, 16, 25, 36, 49, 64, 81, 100, …

$\sqrt{72}$	$\sqrt{80}$	$\sqrt{200}$	$\sqrt{99}$	$\sqrt{48}$	$\sqrt{90}$	$\sqrt{75}$	$\sqrt{98}$
$= \sqrt{36}\sqrt{2}$	$= \sqrt{16}\sqrt{5}$	$= \sqrt{100}\sqrt{2}$	$= \sqrt{9}\sqrt{11}$	$= \sqrt{16}\sqrt{3}$	$= \sqrt{9}\sqrt{10}$	$= \sqrt{25}\sqrt{3}$	$= \sqrt{49}\sqrt{2}$
$= 6\sqrt{2}$	$= 4\sqrt{5}$	$= 10\sqrt{2}$	$= 3\sqrt{11}$	$= 4\sqrt{3}$	$= 3\sqrt{10}$	$= 5\sqrt{3}$	$= 7\sqrt{2}$

9.1 (b) I can simplify surds by multiplying and dividing.

Surds can be **multiplied** and **divided** using the rules:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$\sqrt{2} \times \sqrt{6}$	$\sqrt{5} \times \sqrt{10}$	$\sqrt{\frac{27}{4}}$	$\sqrt{\frac{32}{9}}$	$\frac{\sqrt{28}}{\sqrt{7}}$	$\sqrt{2}(\sqrt{2} - \sqrt{6})$	$\sqrt{3}(\sqrt{6} - \sqrt{3})$
$= \sqrt{12}$	$= \sqrt{50}$	$= \frac{\sqrt{9}\sqrt{3}}{\sqrt{4}}$	$= \frac{\sqrt{16}\sqrt{2}}{\sqrt{9}}$	$= \frac{\sqrt{4}\sqrt{7}}{\sqrt{7}}$	$= \sqrt{4} - \sqrt{12}$	$= \sqrt{18} - \sqrt{9}$
$= \sqrt{4}\sqrt{3}$	$= \sqrt{25}\sqrt{2}$	$= \frac{3\sqrt{3}}{2}$	$= \frac{4\sqrt{2}}{3}$	$= \sqrt{4}$	$= 2 - \sqrt{4}\sqrt{3}$	$= \sqrt{9}\sqrt{2} - 3$
$= 2\sqrt{3}$	$= 5\sqrt{2}$			$= 2$	$= 2 - 2\sqrt{3}$	$= 3\sqrt{2} - 3$

9.1 (c) I can simplify surds by adding and subtracting.

You can only **add** or **subtract** surds when the numbers underneath the square root signs are **equal**.

$\sqrt{50} - \sqrt{32}$	$2\sqrt{45} + \sqrt{80}$	$\sqrt{63} - \sqrt{28} + \sqrt{7}$	$\sqrt{45} + 2\sqrt{20} - 2\sqrt{5}$
$= \sqrt{25}\sqrt{2} - \sqrt{16}\sqrt{2}$	$= 2\sqrt{9}\sqrt{5} + \sqrt{16}\sqrt{5}$	$= \sqrt{9}\sqrt{7} - \sqrt{4}\sqrt{7} + \sqrt{7}$	$= \sqrt{9}\sqrt{5} - 2\sqrt{4}\sqrt{5} - 2\sqrt{5}$
$= 5\sqrt{2} - 4\sqrt{2}$	$= 6\sqrt{5} + 4\sqrt{5}$	$= 3\sqrt{7} - 2\sqrt{7} + \sqrt{7}$	$= 3\sqrt{5} - 4\sqrt{5} + 2\sqrt{5}$
$= \sqrt{2}$	$= 10\sqrt{5}$	$= 2\sqrt{7}$	$= \sqrt{5}$

9.1 (d) I can rationalise the denominator of a fraction.

Rationalising the denominator means converting the denominator so that it is **no longer a surd:**

1. **Multiply** the **numerator** and **denominator** by the **surd**.
2. **Simplify** the fraction if possible.

$\frac{2}{\sqrt{7}}$ $= \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ $= \frac{2\sqrt{7}}{7}$	$\frac{9}{\sqrt{3}}$ $= \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{9\sqrt{3}}{3}$ $= 3\sqrt{3}$	$\frac{3}{\sqrt{6}}$ $= \frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ $= \frac{3\sqrt{6}}{6}$ $= \frac{\sqrt{6}}{2}$	$\frac{6}{\sqrt{12}}$ $= \frac{6}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{6\sqrt{12}}{12}$ $= \frac{6\sqrt{4}\sqrt{3}}{12}$ $= \frac{12\sqrt{3}}{12}$ $= \sqrt{3}$	$\frac{2}{\sqrt{8}}$ $= \frac{2}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}}$ $= \frac{2\sqrt{8}}{8}$ $= \frac{2\sqrt{4}\sqrt{2}}{8}$ $= \frac{4\sqrt{2}}{8}$ $= \frac{\sqrt{2}}{2}$	$\frac{3}{\sqrt{18}}$ $= \frac{3}{\sqrt{18}} \times \frac{\sqrt{18}}{\sqrt{18}}$ $= \frac{3\sqrt{18}}{18}$ $= \frac{3\sqrt{9}\sqrt{2}}{18}$ $= \frac{9\sqrt{2}}{18}$ $= \frac{\sqrt{2}}{2}$

9.2 Indices

9.2 (a) I can solve problems involving the basic rules of indices.

An expression of the form $a \times a \times a \times a$ can be written in **index form** as a^4 - where 4 is called the **index** and a is called the **base**. The following **rules of indices** apply to terms with the **same base**:

$a^0 = 1$	$a^1 = a$	$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$	$(a^m)^n = a^{mn}$

$(2y^3)^2 \times y^3$ $= 4y^6 \times y^3$ $= 4y^9$	$\left(\frac{3}{4}p^5\right)^2$ $= \frac{9}{16}p^{10}$	$k^3(k^4 + k^{-3})$ $= k^7 + k^0$ $= k^7 + 1$	$\frac{3x^4 \times 6x^3}{2x^5}$ $= \frac{18x^7}{2x^5}$ $= 9x^2$	$\frac{9m^6}{3m^4 \times m}$ $= \frac{9m^6}{3m^5}$ $= 3m$	$\frac{10d^5 \times d}{2d^3}$ $= \frac{10d^6}{2d^3}$ $= 5d^3$

9.2 (b) I can solve problems involving negative indices.

You are also expected to be able to work with **negative indices**:

$$a^{-m} = \frac{1}{a^m}$$

$x^2y^5 \div x^{-4}y^7$ $= x^6y^{-2}$ $= \frac{x^6}{y^2}$	$(m^3)^2 \times m^{-12}$ $= m^6 \times m^{-12}$ $= m^{-6}$ $= \frac{1}{m^6}$	$5^0 + 2^{-1}$ $= 1 + \frac{1}{2}$ $= 1\frac{1}{2}$	$(y^4)^3 \div y^{13}$ $= y^{12} \div y^{13}$ $= y^{-1}$ $= \frac{1}{y}$	$7^0 + 4^{-2}$ $= 1 + \frac{1}{16}$ $= 1\frac{1}{16}$	$d^4 \div (d^2)^3$ $= d^4 \div d^6$ $= d^{-2}$ $= \frac{1}{d^2}$

You may be asked to identify a missing index.

$3^n = \frac{1}{9}$ $n = -2$	$5^n = \frac{1}{5}$ $n = -1$	$2^n = \frac{1}{16}$ $n = -4$	$4^n = \frac{1}{64}$ $n = -3$	$(3^2)^n = \frac{1}{81}$ $n = -2$	$(2^{-2})^n = \frac{1}{16}$ $n = 2$

9.2 (c) I can solve problems involving fractional indices.

You are also expected to be able to work with **fractional indices**:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

$a^4 \times \sqrt{a}$ $= a^4 \times a^{\frac{1}{2}}$ $= a^{\frac{9}{2}}$	$(m^3)^2 \div \sqrt{m}$ $= m^6 \div m^{\frac{1}{2}}$ $= m^{\frac{11}{2}}$	$\sqrt{k} \times (k^2)^2$ $= k^{\frac{1}{2}} \times k^4$ $= k^{\frac{9}{2}}$	$\frac{1}{\sqrt{x}}$ $= \frac{1}{x^{\frac{1}{2}}}$ $= x^{-\frac{1}{2}}$	$\frac{1}{\sqrt[3]{y}}$ $= \frac{1}{y^{\frac{1}{3}}}$ $= y^{-\frac{1}{3}}$	$\frac{5}{\sqrt[3]{d}}$ $= \frac{5}{d^{\frac{1}{3}}}$ $= 5d^{-\frac{1}{3}}$

You can also use this to evaluate expressions.

$8^{\frac{2}{3}}$ $= \left(\sqrt[3]{8}\right)^2$ $= 2^2$ $= 4$	$25^{\frac{3}{2}}$ $= \left(\sqrt{25}\right)^3$ $= 5^3$ $= 125$	$16^{\frac{3}{4}}$ $= \left(\sqrt[4]{16}\right)^3$ $= 2^3$ $= 8$	$4^{\frac{3}{2}}$ $= \left(\sqrt{4}\right)^3$ $= 2^3$ $= 8$	$64^{-\frac{1}{3}}$ $= \left(\sqrt[3]{64}\right)^{-1}$ $= 4^{-1}$ $= \frac{1}{4}$	$9^{-\frac{3}{2}}$ $= \left(\sqrt{9}\right)^{-3}$ $= 3^{-3}$ $= \frac{1}{27}$	$32^{-\frac{2}{5}}$ $= \left(\sqrt[5]{32}\right)^{-2}$ $= 2^{-2}$ $= \frac{1}{4}$	$8^{-\frac{4}{3}}$ $= \left(\sqrt[3]{8}\right)^{-4}$ $= 2^{-4}$ $= \frac{1}{16}$

9.3 Scientific Notation

9.3 (a) I can carry out calculations involving scientific notation.

From National 4: You should know the following:

Scientific notation (or **standard form**) is a shorthand way of writing **very large** or **very small** numbers.

In **scientific notation** a number is written as $u \times 10^n$ where $1 \leq u < 10$.

In National 5: You should be able to perform calculations involving **scientific notation**.

Normal Form	32 400 000	59 600	0.000 057 736	0.01353
Scientific Form	3.24×10^7	5.96×10^4	5.7736×10^{-5}	1.353×10^{-2}

There are 5 x 10^9 red blood cells in 1 millilitre of blood.

Calculate in scientific notation the number of red blood cells in 3.25 litres of blood.

Give your answer in scientific notation.

$Cells = 3.25 \times 1000 \times (5 \times 10^9)$

$= 1.625 \times 10^{13}$

The speed of light is approximately 3×10^8 m/s.

The distance from the Sun to Earth is 1.5×10^{11} m. How long does it take for light to travel from the Sun to Earth?

Give your answer in standard form.

$T = (1.5 \times 10^{11}) \div (3 \times 10^8)$

$= 5 \times 10^2$ *seconds*

The time taken for light to reach Earth from the edge of the known universe is 1.4×10^{10} years.

Light travels at the speed of 9.46×10^{12} km/year. Work out the distance, in kilometres.

Give your answer in standard form.

$D = (1.4 \times 10^{10}) \times (9.46 \times 10^{12})$

$= 1.324410^{23}$ *km*

10. Vectors

10.1 Vector Calculations

10.1 (a) I can convert between directed line segments and column vectors.

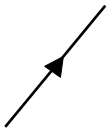
Quantities which have both a **size** and a **direction** are called **vectors**. They may be **represented** in one of two ways:

Column Vectors

In this form **vector** $\begin{pmatrix}4\\1\end{pmatrix}$ has x-component 4 and y-component 1.

Directed Line Segments

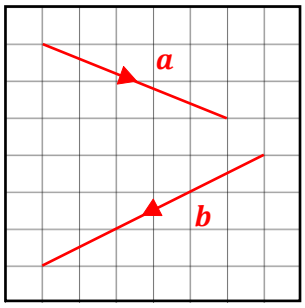
In this form the **vector's size** is shown by the line's **length** and it's **direction** by an **arrow**.



They can also be **named** in one of two ways: 1. A single letter in bold, so for instance $\boldsymbol{a}$.
2. Using the **start** and **endpoints**, so for instance $\overrightarrow{AB}$.

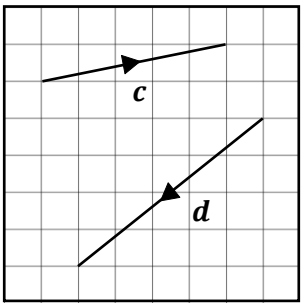
Draw the vectors:

$\boldsymbol{a} = \begin{pmatrix}5\\-2\end{pmatrix}, \boldsymbol{b} = \begin{pmatrix}-6\\-3\end{pmatrix}$.



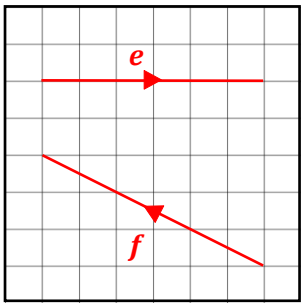
Identify the vectors.

$\boldsymbol{c} = \begin{pmatrix}5\\1\end{pmatrix}, \boldsymbol{d} = \begin{pmatrix}-5\\-4\end{pmatrix}$



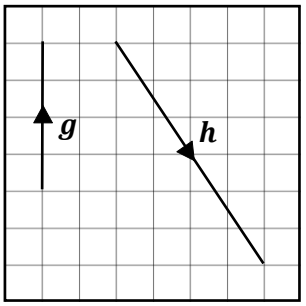
Draw the vectors:

$\boldsymbol{e} = \begin{pmatrix}6\\0\end{pmatrix}, \boldsymbol{f} = \begin{pmatrix}-6\\3\end{pmatrix}$.



Identify the vectors.

$\boldsymbol{g} = \begin{pmatrix}0\\4\end{pmatrix}, \boldsymbol{h} = \begin{pmatrix}4\\-6\end{pmatrix}$



10.1 (b) I can add directed line segments and multiply by scalars.

You should be able to carry out calculations with **vectors** in the form of **directed line segments**.

Multiplying by a Scalar:

A **scalar** is a quantity with **size but no direction**.

You **multiply by a scalar** k by making the vector shorter or longer by a scale factor of k.

Vectors $\boldsymbol{a}$ and $-\boldsymbol{a}$ have the same length but are in **opposite** directions.

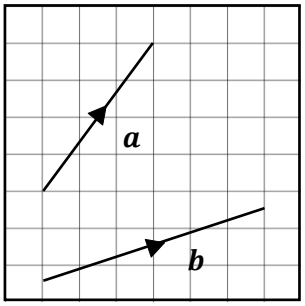
Adding & Subtracting:

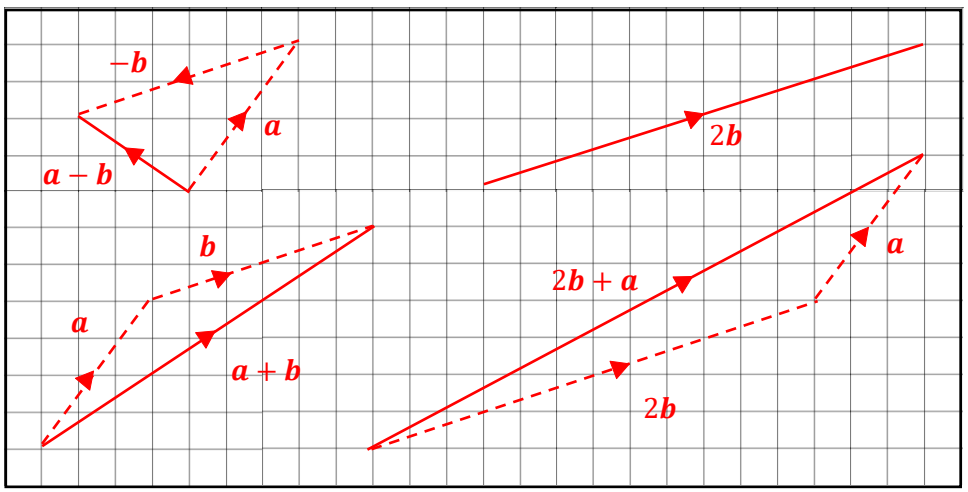
You can **add vectors** to create a **resultant vector** you do this by joining them 'nose to tail'.

It does not matter the order you add vectors so $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$.

Draw the vectors:

- $\boldsymbol{a} + \boldsymbol{b}$
- $\boldsymbol{a} - \boldsymbol{b}$
- $2\boldsymbol{b}$
- $2\boldsymbol{b} + \boldsymbol{a}$





10.1 (c) I can add column vectors and multiply by scalars.

You should also be able to carry out calculations with **vectors** in the form of **column vectors**.

Multiplying by a Scalar:	**Adding & Subtracting:**
Multiply each component of the vector by the scalar.	Add the components of the vectors.

Note: Vectors can be in either two dimensions or three dimensions.

$\boldsymbol{c} = \begin{pmatrix}3\\1\end{pmatrix}$ and $\boldsymbol{d} = \begin{pmatrix}-7\\-2\end{pmatrix}$.

Find $2\boldsymbol{c} - \boldsymbol{d}$.

$= 2\begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}-7\\-2\end{pmatrix}$

$= \begin{pmatrix}6\\2\end{pmatrix} - \begin{pmatrix}-7\\-2\end{pmatrix}$

$= \begin{pmatrix}13\\4\end{pmatrix}$

$\boldsymbol{a} = \begin{pmatrix}2\\-3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}5\\0\end{pmatrix}$.

Find $\boldsymbol{a} + 3\boldsymbol{b}$.

$= \begin{pmatrix}2\\-3\end{pmatrix} + 3\begin{pmatrix}5\\0\end{pmatrix}$

$= \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}15\\0\end{pmatrix}$

$= \begin{pmatrix}17\\-3\end{pmatrix}$

$\boldsymbol{m} = \begin{pmatrix}3\\0\\2\end{pmatrix}$ and $\boldsymbol{n} = \begin{pmatrix}-3\\1\\-3\end{pmatrix}$.

Find $2\boldsymbol{m} - 5\boldsymbol{n}$.

$= 2\begin{pmatrix}3\\0\\2\end{pmatrix} - 5\begin{pmatrix}-3\\1\\-3\end{pmatrix}$

$= \begin{pmatrix}6\\0\\4\end{pmatrix} - \begin{pmatrix}-15\\5\\-15\end{pmatrix}$

$= \begin{pmatrix}21\\-5\\19\end{pmatrix}$

$\boldsymbol{g} = \begin{pmatrix}2\\4\\0\end{pmatrix}$ and $\boldsymbol{h} = \begin{pmatrix}-7\\9\\-5\end{pmatrix}$.

Find $3\boldsymbol{g} - 2\boldsymbol{h}$.

$= 3\begin{pmatrix}2\\4\\0\end{pmatrix} - 2\begin{pmatrix}-7\\9\\-5\end{pmatrix}$

$= \begin{pmatrix}6\\12\\0\end{pmatrix} - \begin{pmatrix}-14\\18\\-10\end{pmatrix}$

$= \begin{pmatrix}20\\-6\\10\end{pmatrix}$

10.2 Magnitude

10.2 (a) I can calculate the magnitude of a vector.

The **length** of a vector $\boldsymbol{a}$ is called its **magnitude**. The **magnitude** of vector $\boldsymbol{a}$ is written as $|\boldsymbol{a}|$.

The **magnitude** of a vector is found using **Pythagoras' Theorem**.

Note: You can also use this idea to calculate the distance between two points in two dimensions or three.

Find $|\boldsymbol{x}|$ if $\boldsymbol{x} = \begin{pmatrix}2\\-5\\1\end{pmatrix}$.

$|\boldsymbol{x}| = \sqrt{2^2 + (-5)^2 + 1^2}$

$= \sqrt{30}$

$= 5.48$

Find $|\boldsymbol{h}|$ if $\boldsymbol{h} = \begin{pmatrix}-1\\9\\6\end{pmatrix}$.

$|\boldsymbol{h}| = \sqrt{(-1)^2 + 9^2 + 6^2}$

$= \sqrt{118}$

$= 10.86$

Calculate the distance between (1, 2) and (4, -2).

$d = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5$

Calculate the distance between (-3, 5) and (8, 0).

$d = \sqrt{11^2 + 5^2}$

$= \sqrt{146}$

$= 12.08$

Calculate the distance between (2, -5, 6) and (-7, -1, 3).

$d = \sqrt{9^2 + 4^2 + 3^2}$

$= \sqrt{25}$

$= 10.30$

Calculate the distance between (1, 0, -5) and (-4, 3, -8).

$d = \sqrt{5^2 + 3^2 + 3^2}$

$= \sqrt{43}$

$= 6.56$

$\boldsymbol{a} = \begin{pmatrix}3\\-4\end{pmatrix}, \boldsymbol{b} = \begin{pmatrix}1\\-2\end{pmatrix}$.

Calculate $|2\boldsymbol{a} - 3\boldsymbol{b}|$.

$2\boldsymbol{a} - 3\boldsymbol{b}$

$= 2\begin{pmatrix}3\\-4\end{pmatrix} - 3\begin{pmatrix}1\\-2\end{pmatrix}$

$= \begin{pmatrix}6\\-8\end{pmatrix} - \begin{pmatrix}3\\-6\end{pmatrix}$

$= \begin{pmatrix}3\\-2\end{pmatrix}$

$|2\boldsymbol{a} - 3\boldsymbol{b}|$

$= \sqrt{3^2 + (-2)^2}$

$= \sqrt{13}$

$= 3.61$

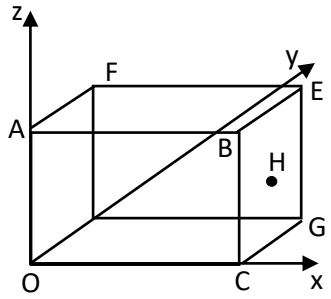
10.3 Vector Pathways

10.3 (a) I can identify coordinates in three dimensions.

You should be able to identify **coordinates** in **three dimensions**.

E has coordinates (6, 4, 3).
H is the centre of CBEG.

Find the other coordinates.



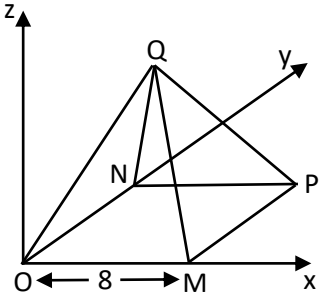
$A\ (0,0,3)$ $F\ (0,4,3)$
$B\ (6,0,3)$ $G\ (6,4,0)$
$C\ (6,0,0)$ $H\ (6,2,1.5)$

Find the length of OE.

$OE = \sqrt{6^2 + 4^2 + 3^2}$
$= \sqrt{61}$
$= 7.81$

The square based pyramid has a height of 9.

Find all the named coordinates.



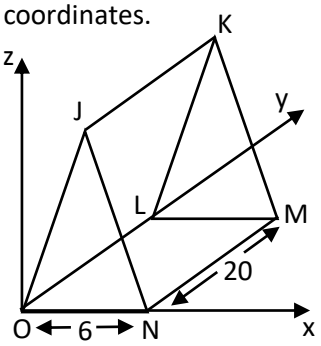
$M\ (8,0,0)$ $Q\ (4,4,9)$
$N\ (0,8,0)$
$P\ (8,8,0)$

Find the length of OQ.

$OQ = \sqrt{4^2 + 4^2 + 9^2}$
$= \sqrt{113}$
$= 10.63$

The triangular prism has a height of 9.

Find all the named coordinates.



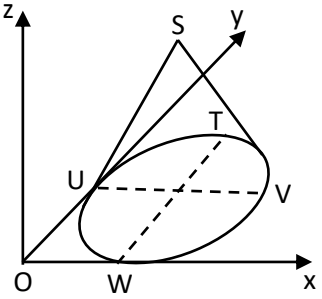
$J\ (3,0,9)$ $M\ (6,20,0)$
$K\ (3,20,9)$ $N\ (6,0,0)$
$L\ (0,20,0)$

Find the length of JM.

$JM = \sqrt{3^2 + 20^2 + 9^2}$
$= \sqrt{490}$
$= 22.14$

The cone has a height of 12 and radius 4.

Find all the named coordinates.



$S\ (4,4,12)$ $V\ (8,4,0)$
$T\ (4,8,0)$ $W\ (4,0,0)$
$U\ (0,4,0)$

Find the length of SV.

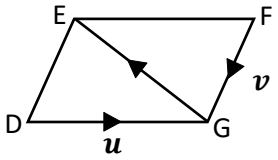
$SV = \sqrt{4^2 + 0^2 + 12^2}$
$= \sqrt{61}$
$= 12.65$

10.3 (b) I can work with vector pathways.

You can express a vector as a combination of other, known, vectors. To do this:

1. Identify a **pathway** from the start to end using other **known vectors**.
2. **Simplify** this if possible.
3. If you move **backwards** along a vector, take the vector away.

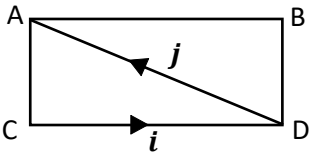
DEFG is a parallelogram.



Find $\overrightarrow{GE}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

$\overrightarrow{GE} = \overrightarrow{GF} + \overrightarrow{FE}$
$= -\boldsymbol{v} - \boldsymbol{u}$

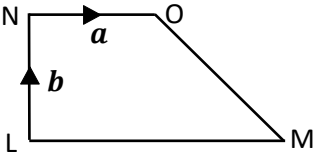
ABCD is a rectangle.



Find $\overrightarrow{BC}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$
$= -\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{i}$
$= -2\boldsymbol{i} - \boldsymbol{j}$

LM is twice the size of NO.



Find $\overrightarrow{OM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\overrightarrow{OM} = \overrightarrow{ON} + \overrightarrow{NL} + \overrightarrow{LM}$
$= -\boldsymbol{a} - \boldsymbol{b} + 2\boldsymbol{a}$
$= \boldsymbol{a} + \boldsymbol{b}$

In triangle DEF,

$\overrightarrow{DF} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$, $\overrightarrow{FE} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Express $\overrightarrow{DE}$ in component form.

$\overrightarrow{DE} = \overrightarrow{DF} + \overrightarrow{FE}$
$= \begin{pmatrix} 5 \\ -8 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
$= \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

11. Trigonometric Skills

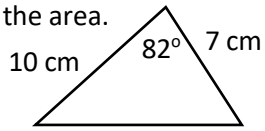
11.1 The Area Rule

11.1 (a) I can use the area rule to calculate the area of a triangle.

If you know **two sides** of a triangle and the **angle between them** you can find its **area.** The formula for this is given in the **exam formulae sheet**:

$$A = \frac{1}{2}ab\sin C$$

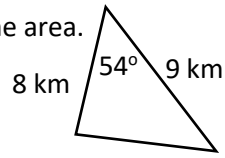
Find the area.



$A_{tri} = \frac{1}{2} \times 7 \times 10 \sin 82^o$

$= 34.66\ cm^2$

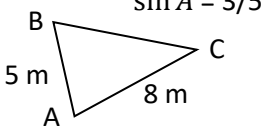
Find the area.



$A_{tri} = \frac{1}{2} \times 9 \times 8 \sin 54^o$

$= 29.12\ km^2$

Find the area.

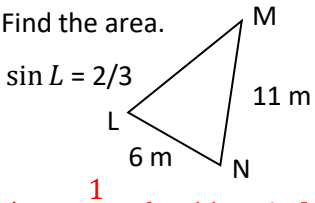


$A_{tri} = \frac{1}{2} \times 5 \times 8 \times \sin A$

$= \frac{1}{2} \times 5 \times 8 \times \frac{3}{5}$

$= 12\ m^2$

Find the area.



$A_{tri} = \frac{1}{2} \times 6 \times 11 \times \sin L$

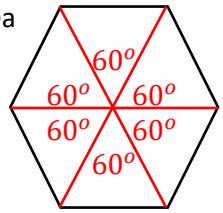
$= \frac{1}{2} \times 6 \times 11 \times \frac{2}{3}$

$= 22\ m^2$

11.1 (b) I can use the area rule to calculate the area of a composite shape.

You can use the **area rule** to calculate the **area of a composite shape**.

Find the area of this hexagon with diameter 10 cm.



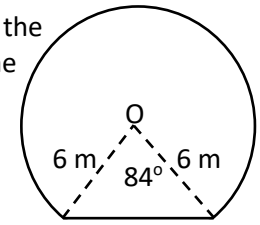
$A_{triangle} = \frac{1}{2} \times 5 \times 5 \sin 60^o$

$= 10.82\ cm^2$

$A_{hexagon} = 6 \times 10.82$

$= 64.92\ cm^2$

Calculate the area of the shape.



$A_{triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 84^o$

$= 17.90\ m^2$

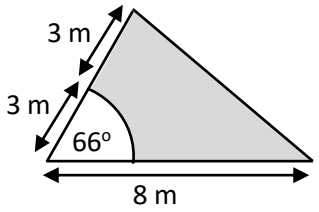
$S.A. = \frac{276}{360} \times \pi \times 6^2$

$= 86.71\ m^2$

$A_{total} = 17.90 + 86.71$

$= 104.61\ m^2$

Calculate the shaded area.



$A_{triangle} = \frac{1}{2} \times 8 \times 6 \times \sin 66^o$

$= 21.93\ m^2$

$S.A. = \frac{66}{360} \times \pi \times 3^2$

$= 5.18\ m^2$

$A_{total} = 21.93 - 5.18$

$= 16.75\ m^2$

11.2 The Sine Rule

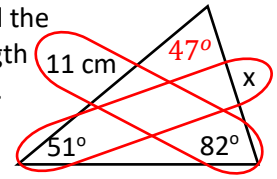
11.2 (a) I can use the sine rule to calculate a missing side.

The **sine rule** states that for a triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This formula is given in the **exam formulae sheet**. This can be used to calculate a **missing side**.

Find the length of x.

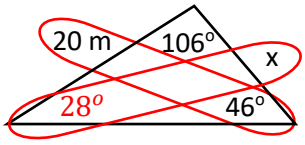


$$\frac{x}{\sin 51^o} = \frac{11}{\sin 82^o}$$

$$x = \frac{11 \sin 51^o}{\sin 82^o}$$

$$= 8.63\ cm$$

Find the length of x.



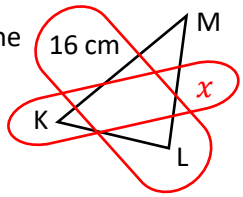
$$\frac{x}{\sin 28^o} = \frac{20}{\sin 46^o}$$

$$x = \frac{20 \sin 28^o}{\sin 46^o}$$

$$= 13.05\ m$$

sin K = 0.6 and sin L = 0.8.

Find the length of LM.



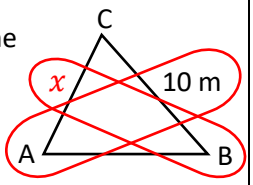
$$\frac{LM}{0.6} = \frac{16}{0.8}$$

$$\frac{LM}{0.6} = 20$$

$$LM = 12\ cm$$

sin A = 0.5 and sin B = 0.4.

Find the length of AC.



$$\frac{AC}{0.4} = \frac{10}{0.5}$$

$$\frac{AC}{0.4} = 20$$

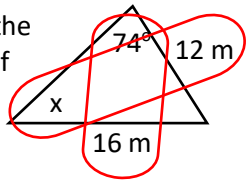
$$AC = 8\ m$$

11.2 (b) I can use the sine rule to calculate a missing angle.

If you are trying to find a **missing angle** the formula can be rearranged to:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Find the size of x.



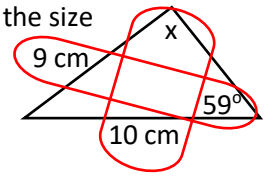
$$\frac{\sin x}{12} = \frac{\sin 74^o}{16}$$

$$\sin x = \frac{12 \sin 74^o}{16}$$

$$x = \sin^{-1}\left(\frac{12 \sin 74^o}{16}\right)$$

$$= 46.13^o$$

Find the size of x.



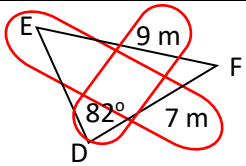
$$\frac{\sin x}{10} = \frac{\sin 59^o}{9}$$

$$\sin x = \frac{10 \sin 59^o}{9}$$

$$x = \sin^{-1}\left(\frac{10 \sin 59^o}{9}\right)$$

$$= 72.25^o$$

Find DEF.



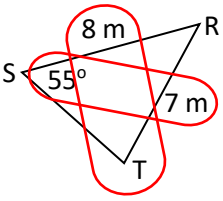
$$\frac{\sin E}{7} = \frac{\sin 82^o}{9}$$

$$\sin E = \frac{7 \sin 82^o}{9}$$

$$E = \sin^{-1}\left(\frac{7 \sin 82^o}{9}\right)$$

$$= 50.37^o$$

Find STR.



$$\frac{\sin T}{8} = \frac{\sin 55^o}{6}$$

$$\sin T = \frac{8 \sin 55^o}{6}$$

$$T = \sin^{-1}\left(\frac{8 \sin 55^o}{6}\right)$$

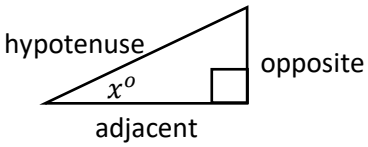
$$= 69.42^o$$

11.2 (c) I can use trigonometry along with the sine rule.

From National 4: You should know the following:

Trigonometry can be used to find sides and angles within a triangle

If one of the angles is called x^o the other sides can be named as shown.



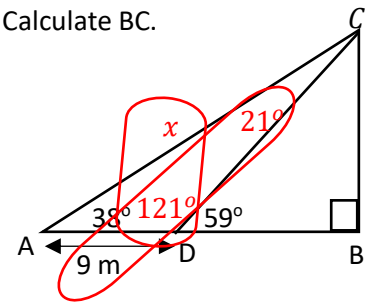
In **Trigonometry** there are three trigonometric ratios you should know:

$$\sin x^o = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos x^o = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan x^o = \frac{\text{opposite}}{\text{adjacent}}$$

This is often shortened down to $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$.

In National 5: You should also be able to answer questions involving **trigonometry** and the **sine rule**.

Calculate BC.



$$\frac{x}{\sin 121^o} = \frac{9}{\sin 21^o}$$

$$x = \frac{9 \sin 121^o}{\sin 21^o}$$

$$= 21.53\ m$$

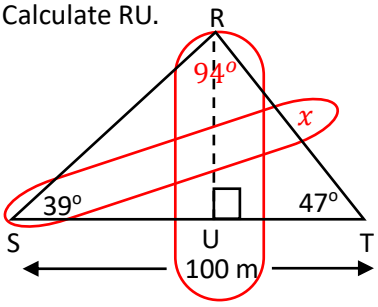
$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

$$\sin 38^o = \frac{BC}{21.53}$$

$$21.53 \sin 38^o = BC$$

$$13.26\ m = BC$$

Calculate RU.



$$\frac{x}{\sin 39^o} = \frac{100}{\sin 94^o}$$

$$x = \frac{100 \sin 39^o}{\sin 94^o}$$

$$= 63.09\ m$$

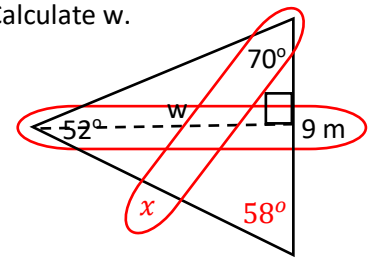
$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

$$\sin 47^o = \frac{RU}{63.09}$$

$$63.09 \sin 47^o = RU$$

$$46.14\ m = RU$$

Calculate w.



$$\frac{x}{\sin 70^o} = \frac{9}{\sin 52^o}$$

$$x = \frac{9 \sin 70^o}{\sin 52^o}$$

$$= 10.73\ m$$

$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

$$\sin 58^o = \frac{w}{10.73}$$

$$10.73 \sin 58^o = w$$

$$9.10\ m = w$$

11.3 The Cosine Rule

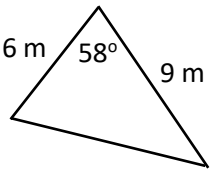
11.3 (a) I can use the cosine rule to calculate a missing side.

To a find a length using the **cosine rule**, you need the **other two sides** and the **angle in between**:

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

this formula is given in the **exam formulae sheet.**

Find the missing length in this diagram.

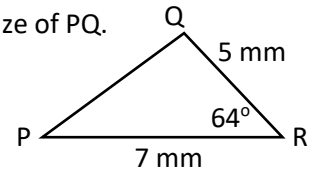


$$a^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \cos 58^o$$

$$a = \sqrt{6^2 + 9^2 - 2 \times 6 \times 9 \cos 58^o}$$

$$= 7.73\ m$$

Find the size of PQ.

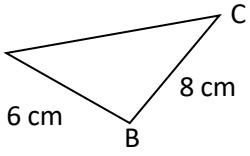


$$x^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \cos 64^o$$

$$x = \sqrt{5^2 + 7^2 - 2 \times 5 \times 7 \cos 64^o}$$

$$= 6.58\ mm$$

If cos B = 0.2 find the length of AC.



$$x^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times 0.2$$

$$x = \sqrt{6^2 + 8^2 - 2 \times 6 \times 8 \times 0.2}$$

$$= 8.99\ cm$$

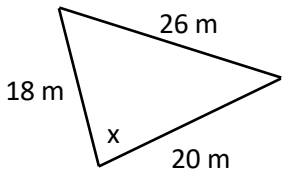
11.3 (b) I can use the cosine rule to calculate a missing angle.

To a find an angle using the **cosine rule**, you must know **all three sides**:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

this formula is given in the **exam formulae sheet.**

Find the size of x.

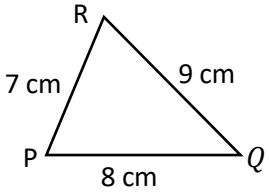


$$\cos x = \frac{18^2 + 20^2 - 26^2}{2 \times 18 \times 20}$$

$$x = \cos^{-1}\left(\frac{18^2 + 20^2 - 26^2}{2 \times 18 \times 20}\right)$$

$$= 86.18^o$$

Calculate the size of PQR.

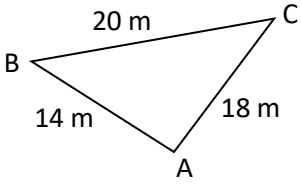


$$\cos PQR = \frac{8^2 + 9^2 - 7^2}{2 \times 8 \times 9}$$

$$PQR = \cos^{-1}\left(\frac{8^2 + 9^2 - 7^2}{2 \times 8 \times 9}\right)$$

$$= 48.19^o$$

Show cos B = 5/21.



$$\cos A = \frac{18^2 + 14^2 - 20^2}{2 \times 18 \times 14}$$

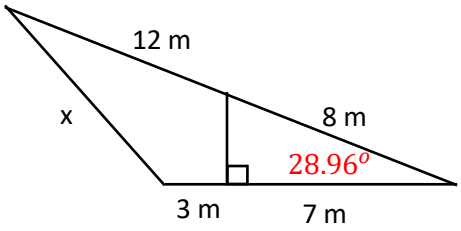
$$= \frac{120}{504}$$

$$= \frac{5}{21}$$

11.3 (c) I can use trigonometry along with the cosine rule.

You should also be able to answer questions involving **trigonometry** and the **cosine rule**.

Find the size of x.



$$S\frac{O}{H}\left(C\frac{A}{H}\right)T\frac{O}{A}$$

$$\cos A = \frac{7}{8}$$

$$A = \cos^{-1}\left(\frac{7}{8}\right)$$

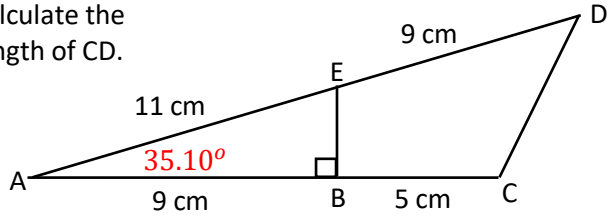
$$= 28.96^o$$

$$x^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \cos 28.96^o$$

$$x = \sqrt{10^2 + 20^2 - 2 \times 10 \times 20 \cos 28.96^o}$$

$$= 12.25\ m$$

Calculate the length of CD.



$$S\frac{O}{H}\left(C\frac{A}{H}\right)T\frac{O}{A}$$

$$\cos A = \frac{9}{11}$$

$$A = \cos^{-1}\left(\frac{9}{11}\right)$$

$$= 35.10^o$$

$$(CD)^2 = 14^2 + 20^2 - 2 \times 14 \times 20 \cos 35.10^o$$

$$CD = \sqrt{14^2 + 20^2 - 2 \times 14 \times 20 \cos 35.10^o}$$

$$= 11.74\ cm$$

11.4 Bearings

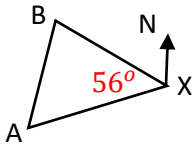
11.4 (a) I can use bearings to solve problems.

Bearings are used to describe the direction in which to travel to get from one place to another.

- They are measured from the **north** line in a **clockwise direction**.
- A bearing can be any angle from 0° to 360° and is written as a **three-figure number**.

Ship A is 9 km from X on a bearing of 256°.

Ship B is 8 km from X on a bearing of 312°.



Calculate the distance between A and B.

$AXB = 312^o - 256^o$

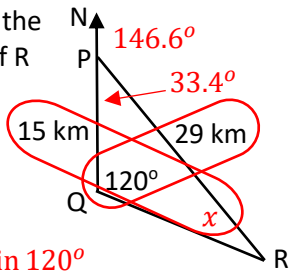
$= 56^o$

$x^2 = 9^2 + 8^2 - 2 \times 9 \times 8 \cos 56^o$

$x = \sqrt{9^2 + 8^2 - 2 \times 9 \times 8 \cos 56^o}$

$= 8.03\ km$

Calculate the bearing of R from P.



$\frac{\sin x}{15} = \frac{\sin 120^o}{29}$

$\sin x = \frac{15 \sin 120^o}{29}$

$x = \sin^{-1}\left(\frac{15 \sin 120^o}{29}\right)$

$= 26.6^o$

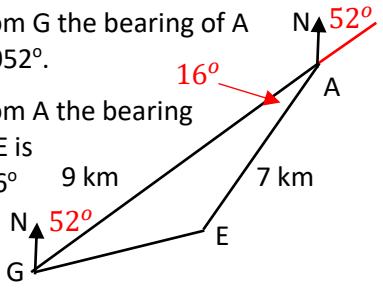
$180^o - 120^o - 26.6^o = 33.4^o$

$Bearing = 180^o - 33.4^o$

$= 146.6^o$

From G the bearing of A is 052°.

From A the bearing of E is 216°



How far apart are G and E.

$GAE = 180^o + 52^o - 216^o$

$= 16^o$

$x^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \cos 16^o$

$x = \sqrt{9^2 + 7^2 - 2 \times 9 \times 7 \cos 16^o}$

$= 6.39\ km$

12. Algebraic Fractions

12.1 Simplifying Fractions

12.1 (a) I can factorise and simplify algebraic fractions.

To simplify an **algebraic fraction** you must first **factorise** both the **numerator** and the **denominator**.

$\frac{m^2-9}{3m+9}$. $= \frac{(m+3)(m-3)}{3(m+3)}$ $= \frac{m-3}{3}$	$\frac{k^2+2k}{k^2+3k+2}$. $= \frac{k(k+2)}{(k+1)(k+2)}$ $= \frac{k}{k+1}$	$\frac{x^2+4x+3}{x^2-1}$. $= \frac{(x+1)(x+3)}{(x-1)(x+1)}$ $= \frac{x+3}{x-1}$	$\frac{d^2-2d-15}{d^2+6d+9}$. $= \frac{(d+3)(d-5)}{(d+3)(d+3)}$ $= \frac{d-5}{d+3}$

12.2 Algebraic Fractions

12.2 (a) I can multiply two algebraic fractions.

You should be able to **multiply** and **simplify** two **algebraic fractions**.

$\frac{x^2}{12} \times \frac{8}{x}$. $= \frac{x^2}{12} \times \frac{8}{x}$ $= \frac{2x}{3}$	$\frac{10y^2}{7x} \times \frac{14x^2}{6y}$. $= \frac{10y^2}{7x} \times \frac{14x^2}{6y}$ $= \frac{10xy}{3}$	$\frac{10m^2}{9n} \times \frac{n}{2m}$. $= \frac{10m^2}{9n} \times \frac{n}{2m}$ $= \frac{5m}{9}$	$\frac{5g}{9h^2} \times \frac{3gh}{2}$. $= \frac{5g}{9h^2} \times \frac{3gh}{2}$ $= \frac{5gh}{6h} = \frac{5g}{6}$	$\frac{x^2}{y^4} \times \frac{y^3}{x^5}$. $= \frac{x^2}{y^3} \times \frac{y^3}{x^5}$ $= \frac{y}{x^3}$

12.2 (b) I can divide one algebraic fraction by another.

You should also be able to **divide** and **simplify** two **algebraic fractions**.

$\frac{5y}{8x} \div \frac{9y}{4x}$. $= \frac{5y}{8x} \times \frac{4x}{9y}$ $= \frac{5}{18}$	$\frac{10m}{ab} \div \frac{5m}{ak}$. $= \frac{10m}{ab} \times \frac{ak}{5m}$ $= \frac{2k}{b}$	$\frac{n}{4m} \div \frac{3n}{m^2}$. $= \frac{n}{4m} \times \frac{m^2}{3n}$ $= \frac{m}{12}$	$\frac{2x}{y^2} \div \frac{x}{3y}$. $= \frac{2x}{y^2} \times \frac{3y}{x}$ $= \frac{6}{y}$	$\frac{c^3}{d^5} \div \frac{2c}{d^7}$. $= \frac{c^3}{d^5} \times \frac{d^7}{2c}$ $= \frac{c^2d^2}{2}$

12.2 (c) I can add and subtract two algebraic fractions.

You should also be able to **add** and **subtract** two algebraic fractions including fractions with brackets.

- You should multiply out the brackets in the **numerator** and simplify.
- The **denominator** meanwhile is normally left in **factor form**.

$\frac{2}{m}+\frac{3}{m^2}$. $=\frac{2m}{m^2}+\frac{3}{m^2}$ $=\frac{2m+3}{m^2}$	$\frac{8a}{3b}+\frac{3}{b^2}$. $=\frac{8ab}{3b^2}+\frac{9}{3b^2}$ $=\frac{8ab+9}{3b^2}$	$\frac{x+1}{2}+\frac{x-3}{3}$. $=\frac{3(x+1)}{6}+\frac{2(x-3)}{6}$ $=\frac{3x+3+2x-6}{6}$ $=\frac{5x-3}{6}$	$\frac{5}{y+2}-\frac{2}{y-3}$. $=\frac{5(y-3)}{(y+2)(y-3)}$ $-\frac{2(y+2)}{(y+2)(y-3)}$ $=\frac{5y-15-2y-4}{(y+2)(y-3)}$ $=\frac{3y-19}{(y+2)(y-3)}$