National 5 Applications of Mathematics

Course Notes & Solutions

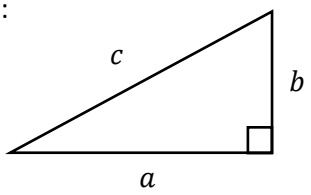
These notes are designed as a revision aid to the National 5 Apps course, they divide the course into 90 individual lessons. This document should link directly to the PowerPoint 'National 5 Apps Course' as well as being backed up by the PowerPoints 'National 5 Apps Exam Questions – by Topic', 'National 5 Apps Exam Questions – by Paper' and 'National 5 Apps Starter Questions'.

FORMULAE LIST

Circumference of a circle: $C = \pi d$

Area of a circle: $A = \pi r^2$

Theorem of Pythagoras:



$a^2 + b^2 = c^2$

Volume of a cylinder: $V = \pi r^2 h$

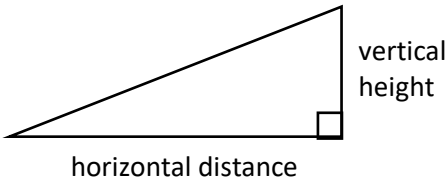
Volume of a prism: $V = Ah$

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Standard deviation: $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n - 1}}$, where n is the sample size.

Gradient:



$$\text{gradient} = \frac{\text{vertical height}}{\text{horizontal distance}}$$

National 5 Apps Learning Intentions

Unit One

1. Fractions & Percentages (11 Lessons)			
1.1 Decimals	(a) I can round to a given number of significant figures.	5	1 2 3 4 5
	(b) I can carry out the four basic operations with integers.	5	1 2 3 4 5
	(c) I can carry out the four basic operations with decimals.	5	1 2 3 4 5
	(d) I can use BIDMAS with decimals.	6	1 2 3 4 5
1.2 Percentages	(a) I can calculate a percentage with or without a calculator.	6	1 2 3 4 5
	(b) I can express one number as a percentage of another.	7	1 2 3 4 5
1.3 Fractions	(a) I can calculate a fraction of a quantity and simplify fractions.	7	1 2 3 4 5
	(b) I can compare the sizes of two fractions.	8	1 2 3 4 5
	(c) I can add and subtract fractions.	8	1 2 3 4 5
	(d) I can convert between top heavy fractions and mixed numbers.	9	1 2 3 4 5
1.4 Conversions	(a) I can convert between decimals, fractions and percentages.	9	1 2 3 4 5

2. Distance & Time (4 Lessons)			
2.1 Time Intervals	(a) I can calculate time intervals using both 12 and 24-hour time.	11	1 2 3 4 5
	(b) I can work with time intervals and different time zones.	11	1 2 3 4 5
2.2 Distance & Speed	(a) I can convert between different units of time.	11	1 2 3 4 5
	(b) I can calculate distance, speed and time.	12	1 2 3 4 5

3. Tables & Scale (7 Lessons)			
3.1 Reading Tables	(a) I can read and interpret information displayed in a table.	13	1 2 3 4 5
3.2 Precedence Tables	(a) I can read and construct precedence tables.	13	1 2 3 4 5
	(b) I can make decisions based on precedence tables.	14	1 2 3 4 5
3.3 Scale Drawings	(a) I can read values from a scale.	14	1 2 3 4 5
	(b) I can mark values down on a scale.	15	1 2 3 4 5
	(c) I can make a scale drawing using a protractor.	15	1 2 3 4 5
	(d) I can make a scale drawing using bearings.	15	1 2 3 4 5

4. Money Problems (8 Lessons)			
4.1 Appreciation	(a) I can carry out calculations involving appreciation and depreciation.	17	1 2 3 4 5
	(b) I can work with more challenging appreciation questions.	17	1 2 3 4 5
	(c) I can work with questions involving multiple percentage changes.	17	1 2 3 4 5
4.2 Shares	(a) I can calculate profit and loss.	18	1 2 3 4 5
	(b) I can perform calculations involving buying and selling shares.	18	1 2 3 4 5
4.3 Foreign Exchange	(a) I can convert between different currencies.	19	1 2 3 4 5
	(b) I can convert between multiple different currencies.	19	1 2 3 4 5
	(c) I can find the foreign exchange rate of two currencies.	19	1 2 3 4 5

Unit Two

5. Proportion & Probability (9 Lessons)			
5.1 Ratio Calculations	(a) I can share a total in a given ratio.	20	1 2 3 4 5
	(b) I can use a given ratio to solve a problem.	20	1 2 3 4 5
5.2 Direct Proportion	(a) I can use direct proportion to solve a problem.	20	1 2 3 4 5
	(b) I can convert between different types of units.	20	1 2 3 4 5
5.3 Inverse Proportion	(a) I can use inverse proportion to solve a problem.	21	1 2 3 4 5
5.4 Probability	(a) I can calculate the probability of a single event happening.	21	1 2 3 4 5
	(b) I can compare two probabilities as fractions or decimals.	22	1 2 3 4 5
	(c) I can make decisions based on expected frequencies.	22	1 2 3 4 5
	(d) I can calculate the probability of combined events.	22	1 2 3 4 5

6. 2D Shapes (8 Lessons)			
6.1 Perimeter	(a) I can calculate the perimeter of a quadrilateral.	23	1 2 3 4 5
	(b) I can calculate the perimeter of a triangle.	23	1 2 3 4 5
	(c) I can calculate the circumference of a circle.	23	1 2 3 4 5
	(d) I can calculate the perimeter of a composite shape.	23	1 2 3 4 5
6.2 Area	(a) I can calculate the area of a rectangle.	24	1 2 3 4 5
	(b) I can calculate the area of a triangle.	24	1 2 3 4 5
	(c) I can calculate the area of a circle.	25	1 2 3 4 5
	(d) I can calculate the area of a composite shape.	25	1 2 3 4 5

7. Graphs & Charts (7 Lessons)			
7.1 Reading Diagrams	(a) I can read and interpret information displayed in a bar chart.	26	1 2 3 4 5
	(b) I can read and interpret information displayed in a line graph.	26	1 2 3 4 5
	(c) I can construct and interpret stem-and-leaf diagrams.	26	1 2 3 4 5
	(d) I can construct and interpret back to back stem-and-leaf diagrams.	27	1 2 3 4 5
7.2 Pie Charts	(a) I can interpret information displayed in a pie chart.	27	1 2 3 4 5
	(b) I can construct a pie chart given the relevant information.	27	1 2 3 4 5
7.3 Scattergraphs	(a) I can plot and use a line of best fit.	28	1 2 3 4 5

8. Budgeting (6 Lessons)			
8.1 Budgets	(a) I can carry out calculations involving a budget.	29	1 2 3 4 5
	(b) I can make decisions based on a budget.	29	1 2 3 4 5
8.2 Best Buys	(a) I can decide on the best deal to buy an item.	29	1 2 3 4 5
8.3 Payment Plans	(a) I can calculate the cost of an item bought by hire purchase.	30	1 2 3 4 5
	(b) I can work out how much is to be repaid after borrowing money.	30	1 2 3 4 5
	(c) I can calculate how much insurance will cost.	31	1 2 3 4 5

Unit Three

9. 3D Shapes (9 Lessons)			
9.1 Formulae	(a) I can work with formulae expressed in words.	32	1 2 3 4 5
	(b) I can work with formulae expressed in symbols.	32	1 2 3 4 5
9.2 Volume	(a) I can calculate the volume of a cube or a cuboid.	32	1 2 3 4 5
	(b) I can calculate the volume of basic three-dimensional shapes.	33	1 2 3 4 5
	(c) I can use the volume of a shape to calculate its height.	33	1 2 3 4 5
	(d) I can calculate the volume of a prism.	33	1 2 3 4 5
	(e) I can calculate the volume of composite shapes.	34	1 2 3 4 5
9.3 Packing Problems	(a) I can solve problems involving packing containers.	34	1 2 3 4 5
	(b) I can solve harder problems involving packing containers.	35	1 2 3 4 5

10. Measures of Spread (9 Lessons)			
10.1 Tolerance	(a) I can work with tolerance notation and intervals.	36	1 2 3 4 5
	(b) I can work with tolerance notation involving percentages.	36	1 2 3 4 5
10.2 Averages	(a) I can find the mean, median, mode and range of a data set.	37	1 2 3 4 5
10.3 Box Plots	(a) I can find the quartiles of a data set.	37	1 2 3 4 5
	(b) I can find the interquartile and semi-interquartile ranges.	37	1 2 3 4 5
	(c) I can compare the median and interquartile ranges of data sets.	38	1 2 3 4 5
	(d) I can construct a box plot to represent a data set.	38	1 2 3 4 5
10.4 Standard Deviation	(a) I can calculate the standard deviation of a data set.	39	1 2 3 4 5
	(b) I can compare the mean and standard deviations of two data sets.	39	1 2 3 4 5

11. Straight Lines (5 Lessons)			
11.1 Gradient	(a) I can identify the gradient of a straight line as a fraction.	40	1 2 3 4 5
	(b) I can identify the gradient of a straight line as a decimal.	40	1 2 3 4 5
	(c) I can identify the gradient of a line from its coordinates.	41	1 2 3 4 5
11.2 Pythagoras Theorem	(a) I can use Pythagoras' theorem to calculate the hypotenuse.	41	1 2 3 4 5
	(b) I can use Pythagoras' theorem to calculate a shorter side.	41	1 2 3 4 5

12. Pay Slips (7 Lessons)			
12.1 Gross Pay	(a) I can carry out calculations based on wages and salaries.	42	1 2 3 4 5
	(b) I can calculate a wage involving overtime pay.	42	1 2 3 4 5
	(c) I can calculate a wage involving commission.	42	1 2 3 4 5
12.2 Net Pay	(a) I can calculate net pay given gross pay and deductions.	43	1 2 3 4 5
	(b) I can solve problems involving net pay.	43	1 2 3 4 5
12.3 Deductions	(a) I can calculate how much Income Tax a person should pay.	44	1 2 3 4 5
	(b) I can calculate how much National Insurance a person should pay.	44	1 2 3 4 5

1. Fractions & Percentages

1.1 Decimals

1.1 (a) I can round to a given number of significant figures.

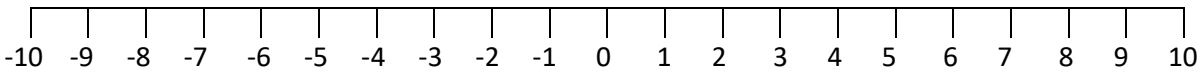
You should be able to round a given number to a required number of **significant figures**.

Number	7 462	59 357	43.573	2.86521	0.024 781	0.007 438	0.000 262 8
1 sig. fig.	7 000	60 000	40	3	0.02	0.007	0.000 3
2 sig. fig.	7 500	59 000	44	2.9	0.025	0.007 4	0.000 26
3 sig. fig.	7 460	59 400	43.6	2.87	0.024 8	0.007 44	0.000 263

1.1 (b) I can carry out the four basic operations with integers.

An **integer** is a **whole number** (not a fractional number) that can be positive, negative, or zero.

To **add** and **subtract integers** you can use the **number line**.



- To **add** move **right** and to **subtract** move **left.**
- If you **subtract** a **negative number** this is the same as **adding**.

The temperature is 11°C in London and -3°C in Aberdeen. Calculate the difference. $11 - (-3) = 11 + 3$ $= 14^oC$	The temperature in Belfast is 4°C. In Dublin it is 7°C less. What is the temperature in Dublin? $4 - 7 = -3^oC$	The temperature is 24°C in Madrid and -8°C in Toronto. Calculate the difference. $24 - (-8) = 24 + 8$ $= 32^oC$	The temperature in Alaska fell 10°C one night. If it fell to -8°C what was the original temperature. $-8 + 10 = 2^oC$

To **multiply** and **divide integers** you can use the facts.

- A **positive** multiplied (or divided) by a **negative** gives a **negative**.
- **Two positives** multiplied (or divided) give a **positive** as do **two negatives**.

$8 \times (-7)$ $= -56$	-4×6 $= -24$	$-2 \times (-7)$ $= 14$	$15 \div (-5)$ $= -3$	$-40 \div (-5)$ $= 8$	$-81 \div 9$ $= -9$	$-64 \div (-8)$ $= 8$

1.1 (c) I can carry out the four basic operations with decimals.

You should be able to **add**, **subtract**, **multiply** and **divide decimals**.

$4.23 + 3.183$ $= 7.413$	$15.1 - 2.77$ $= 12.33$	7.42×9 $= 66.78$	$13.8 \div 8$ $= 1.725$	5.28×80 $= 42.24 \times 10$ $= 422.4$	$522 \div 30$ $= 174 \div 10$ $= 17.4$	$2 \div 7$ $= 0.285\ 7$ $= 0.286$

Questions involving these will usually involve a context.

Bryan has £18.60 and is given £5 from his Gran and £10 from his Uncle. Bryan spends £17.80 at the cinema. How much money does he now have? $18.60 + 5 + 10 - 17.80$ $= 23.60 + 10 - 17.80$ $= 33.60 - 17.80$ $= £15.80$	Gravel is sold in 12 kilogram bags each costing £46.40. Jane needs 100 kilograms of gravel for her driveway. How much will she have to pay? $100 \div 12 = 50 \div 6$ $= 8.33$ $\approx 9\ bags$ $9 \times 46.40 = £417.60$	Coal is bought in two sizes of bags. A small 8 kg bag costs £21.20 while a larger 30 kg bag costs £76.20. Which bag is better value? $21.20 \div 8 = £2.65$ $76.20 \div 30 = 7.62 \div 3$ $= £2.54$ *The 30 kg bag is better value as it costs 11 p less per kg.*	Fencing is sold in lengths 6 metres long costing £11.70. Amy needs 110 metres of fencing for her garden. How much will she have to pay? $110 \div 6 = 50 \div 6$ $= 18.33$ $\approx 19\ lengths$ $19 \times 11.70 = £222.30$

1.1 (d) I can use BIDMAS with decimals.

BIDMAS is an acronym used to help you remember the **order of operations**.

BRACKETS INDICES DIVISION MULTIPLICATION ADDITION SUBTRACTION

$52 - 5.4 \times 8 + 3.9$ $= 52 - 43.2 + 3.9$ $= 8.8 + 3.9$ $= 12.7$	$1.3 + (2.2 + 4.81) \times 3$ $= 1.3 + 7.01 \times 3$ $= 1.3 + 21.03$ $= 22.33$	$7.9 - (5.2 + 1.4) \div 3$ $= 7.9 - 6.6 \div 3$ $= 7.9 - 2.2$ $= 5.7$	$9.4 + 6.4 \div 8 - 4.3$ $= 9.4 + 0.8 - 4.3$ $= 10.2 - 4.3$ $= 5.9$

1.2 Percentages

1.2 (a) I can calculate a percentage with or without a calculator.

To calculate a **percentage** of a total using a **calculator:**

1. Divide the given percentage by 100.
2. Multiply by the total.

James uses 70% of his full tank of petrol driving to Stranraer. His car holds 60 litres of petrol. How much is left in the tank? $70 \div 100 \times 60 = 42$ $60 - 42 = 18\ litres$	Bob buys a new stereo. The price on the label is £300, but he must add 20% VAT onto this. How much would he pay? $20 \div 100 \times 300 = 60$ $300 + 60 = £360$	A shop is having a sale and is offering a 30% discount on all goods. How much would you pay for an item worth £140? $30 \div 100 \times 140 = 42$ $140 - 42 = £98$	Susan is paid £300 per week plus 2% commission on her sales. How much is she paid in a week where her sales are £4 500? $2 \div 100 \times 4\,500 = 90$ $300 + 90 = £390$

Finding a **percentage without using a calculator** requires a little more work.

To find 1% divide by 100.	To find 10% divide by 10.	To find 25% divide by 4.

You can make any percentage required by combining the three facts above.

James buys a new laptop. The original price was £600. The shop is having a sale with 20% off. How much will James pay? $10\%\ of\ 600 = 60$ $20\%\ of\ 600 = 2 \times 60$ $= 120$ $600 - 120 = £480$	Of 220 pupils in first year in Bathgate Academy, 45% are boys. How many boys are in this first year cohort? $10\%\ of\ 220 = 22$ $20\%\ of\ 220 = 2 \times 22$ $= 44$ $25\%\ of\ 220 = 55$ $45\%\ of\ 220 = 44 + 55$ $= 99\ boys$	In a test 10% of the pupils failed. Of those who passed 60% got Cs and Ds. If 50 pupils sat the test how many got As or Bs? $10\%\ of\ 50 = 5$ $50 - 5 = 45$ $10\%\ of\ 45 = 4.5$ $60\%\ of\ 45 = 6 \times 4.5$ $= 27$ $45 - 27 = 18\ pupils$	James bought a clock for £700. The value of the clock has increased by 15% since then. How much is it now worth? $10\%\ of\ 700 = 70$ $5\%\ of\ 700 = 70 \div 2$ $= 35$ $15\%\ of\ 700 = 70 + 35$ $= £105$

1.2 (b) I can express one number as a percentage of another.

You can express one number as a percentage of another by dividing one number by the other and multiplying by 100. This can be used to calculate a **percentage increase** or **decrease** as well.

A tree grew in one year from 150 cm to 180 cm. Calculate the percentage increase in the tree's height. $180 - 150 = 30$ $30 \div 150 = 0.2$ $0.2 \times 100 = 20\%$	A car costs £12 000 and is sold for £9 600. Calculate the percentage decrease in the car's value. $12\,000 - 9\,600 = 2\,400$ $2\,400 \div 12\,000 = 0.2$ $0.2 \times 100 = 20\%$	Erin's rent is increasing from £540 to £594 a month. Calculate the percentage increase in her rent. $594 - 540 = 54$ $54 \div 540 = 0.1$ $0.1 \times 100 = 10\%$	One month a hotel has 400 visitors the next month 412. Calculate the percentage increase in visitors. $412 - 400 = 12$ $12 \div 400 = 0.03$ $0.03 \times 100 = 3\%$

1.3 Fractions

1.3 (a) I can calculate a fraction of a quantity and simplify fractions.

To work with fractions, you should know both how to find a fraction of a quantity and how to simplify them.

$\frac{3}{5}$ of 850. $= 850 \div 5 \times 3$ $= 170 \times 3$ $= 510$	$\frac{7}{8}$ of 96. $= 96 \div 8 \times 7$ $= 12 \times 7$ $= 84$	$\frac{3}{8}$ of 256. $= 256 \div 8 \times 3$ $= 32 \times 3$ $= 96$	$\frac{3}{4}$ of 70. $= 70 \div 4 \times 3$ $= 17.5 \times 3$ $= 52.5$	$\frac{5}{6}$ of 72. $= 72 \div 6 \times 5$ $= 12 \times 5$ $= 60$	$\frac{2}{5}$ of 12. $= 12 \div 5 \times 2$ $= 2.4 \times 2$ $= 4.8$

Fraction	$\frac{12}{16}$	$\frac{24}{48}$	$\frac{15}{25}$	$\frac{18}{24}$	$\frac{27}{30}$	$\frac{12}{40}$	$\frac{12}{60}$	$\frac{28}{63}$	$\frac{16}{70}$	$\frac{2}{84}$	$\frac{32}{54}$
Simplified	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{9}{10}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{4}{9}$	$\frac{8}{35}$	$\frac{1}{42}$	$\frac{16}{27}$

1.3 (b) I can compare the sizes of two fractions.

To compare the size of two fractions you need them to have either the **same numerator** or the **same denominator**.

- If two fractions have the **same numerator** the one with the **larger denominator** will be **smaller.**
- If two fractions have the **same denominator** the one with the **larger numerator** will be **larger.**

Euan has used $\frac{2}{7}$ of his data.

James has used spent $\frac{6}{19}$ of his.

Who has used a greater fraction of their data?

Euan: $\frac{2}{7} = \frac{6}{21}$

James: $\frac{6}{19}$

James used a greater fraction as $\frac{6}{19} > \frac{6}{21}$.

Eilidh has set herself a spending allowance of £45 and spends £15 of this. May has set herself a spending allowance of £60 and spends £18 of this. Who has used a greater proportion?

Eilidh: $\frac{15}{45} = \frac{1}{3} = \frac{3}{9}$

May: $\frac{18}{60} = \frac{3}{10}$

Eilidh used a greater proportion as $\frac{3}{9} > \frac{3}{10}$.

David spent $\frac{3}{5}$ of his money.

Kirsty spent $\frac{5}{9}$ of hers.

Who spent a greater fraction of their pocket money?

David: $\frac{3}{5} = \frac{27}{45}$

Kirsty: $\frac{5}{9} = \frac{25}{45}$

David spent a greater fraction as $\frac{27}{45} > \frac{25}{45}$.

Hugh has used 4 GB out of his monthly limit of 12 GB. Anne meanwhile has used 7 GB out of her monthly limit of 30 GB. Who has used a greater proportion of their monthly limit?

Hugh: $\frac{4}{12} = \frac{1}{3} = \frac{10}{30}$

Anne: $\frac{7}{30}$

Hugh used a greater proportion as $\frac{10}{30} > \frac{7}{30}$.

1.3 (c) I can add and subtract fractions.

To **add** or **subtract** two fractions:

1. Make sure the **denominators** are the **same**.
2. Add or subtract the two **numerators**.
3. Make sure to **simplify** the fraction if possible.

A group of people were asked their favourite primary colour.

- $\frac{1}{4}$ chose red
- $\frac{2}{5}$ chose blue
- The rest chose yellow.

Calculate the fraction of pupils who chose yellow.

A group of people were asked how they got to work.

- $\frac{1}{5}$ cycled
- $\frac{2}{3}$ drove
- The rest took the bus to work.

Calculate the fraction of people taking the bus.

The marks in an English class test are listed:

- $\frac{1}{4}$ got a B
- $\frac{1}{3}$ got a C
- $\frac{1}{6}$ failed
- The rest got A's.

Calculate the fraction of the class getting an A.

A box contains pieces of fruit, of these:

- $\frac{2}{5}$ were apples
- $\frac{1}{3}$ were bananas
- $\frac{1}{6}$ were oranges
- The rest were pears.

Calculate the fraction which were pears.

$\frac{1}{4}+\frac{2}{5}=\frac{5}{20}+\frac{8}{20}$ $=\frac{13}{20}$ $\frac{7}{20}$ *chose yellow.*	$\frac{1}{5}+\frac{2}{3}=\frac{3}{15}+\frac{10}{15}$ $=\frac{13}{15}$ $\frac{2}{15}$ *took the bus.*	$\frac{1}{4}+\frac{1}{3}+\frac{1}{6}$ $=\frac{3}{12}+\frac{4}{12}+\frac{2}{12}$ $=\frac{9}{12}$ $=\frac{3}{4}$ $\frac{1}{4}$ *got A's.*	$\frac{2}{5}+\frac{1}{3}+\frac{1}{6}$ $=\frac{12}{30}+\frac{10}{30}+\frac{5}{30}$ $=\frac{27}{30}$ $=\frac{9}{10}$ $\frac{1}{10}$ *were pears.*

1.3 (b) I can convert between top heavy-fractions and mixed numbers.

A **top-heavy fraction** is one where the numerator is greater than the denominator.

You should be able to change a **top-heavy fraction** to a **mixed number**.

$\frac{12}{5}$	$\frac{25}{6}$	$\frac{30}{7}$	$\frac{50}{9}$	$\frac{32}{5}$	$\frac{44}{7}$	$\frac{47}{9}$	$\frac{19}{5}$	$\frac{55}{8}$	$\frac{35}{6}$	$\frac{60}{7}$	$\frac{32}{9}$
$2\frac{2}{5}$	$4\frac{1}{6}$	$4\frac{2}{7}$	$5\frac{5}{9}$	$6\frac{2}{5}$	$6\frac{2}{7}$	$5\frac{2}{9}$	$3\frac{4}{5}$	$6\frac{7}{8}$	$5\frac{5}{6}$	$8\frac{4}{7}$	$3\frac{5}{9}$

You should also be able to work the other way around and change a **mixed number** to a **top-heavy fraction**.

$2\frac{1}{3}$	$4\frac{3}{5}$	$7\frac{3}{4}$	$3\frac{2}{9}$	$1\frac{3}{10}$	$2\frac{2}{7}$	$1\frac{7}{9}$	$5\frac{4}{5}$	$2\frac{7}{8}$	$9\frac{5}{6}$	$3\frac{4}{7}$	$5\frac{3}{9}$
$\frac{7}{3}$	$\frac{23}{5}$	$\frac{31}{4}$	$\frac{29}{5}$	$\frac{13}{10}$	$\frac{16}{7}$	$\frac{16}{9}$	$\frac{29}{5}$	$\frac{23}{8}$	$\frac{59}{6}$	$\frac{25}{7}$	$\frac{48}{9}$

1.4 Conversions

1.4 (a) I can convert between decimals, fractions and percentages.

You should be able to convert between **decimals**, **fractions** and **percentages**.

Fraction	→	Divide the numerator by the denominator and multiply by 100.	→	**Percentage**
	←	Put over 100 and simplify.	←	

Fraction	→	Divide the numerator by the denominator.	→	**Decimal**
	←	Put over 10 or 100 and simplify.	←	

Decimal	→	Multiply by 100 and add the percentage (%) symbol.	→	**Percentage**
	←	Divide by 100 and remove the percentage (%) symbol.	←	

Decimal	0.7	0.15	0.8	0.52	0.08	0.99	0.003	0.46	0.85
Percentage	70%	15%	80%	52%	8%	99%	0.3%	46%	85%
Fraction	$\frac{7}{10}$	$\frac{3}{20}$	$\frac{4}{5}$	$\frac{26}{50}$	$\frac{2}{25}$	$\frac{99}{100}$	$\frac{3}{1\,000}$	$\frac{23}{50}$	$\frac{17}{20}$
Working (if needed)									

Converting between the three can allow you to order different numbers in terms of size.

Write the following in order from smallest to largest.	Write the following in order from greatest to least.	Write the following in order from least to greatest.	Write the following in order from greatest to least.
0.488, $\frac{4}{9}$, 42.4%, 0.4	36%, $\frac{1}{3}$, 35.5%, 0.323	0.7, $\frac{5}{8}$, 68.8%, $\frac{2}{3}$	0.8, $\frac{3}{4}$, 77%, $\frac{7}{8}$
$* \ 0.488$	$* \ 0.36$	$* \ 0.7$	$* \ 0.8$
$* \ \frac{4}{9} = 0.444$	$* \ \frac{1}{3} = 0.333$	$* \ \frac{5}{8} = 0.625$	$* \ \frac{3}{4} = 0.75$
$* \ 42.4\% = 0.424$	$* \ 35.5\% = 0.355$	$* \ 68.8\% = 0.688$	$* \ 77\% = 0.77$
$* \ 0.4$	$* \ 0.323$	$* \ \frac{2}{3} = 0.667$	$* \ \frac{7}{8} = 0.875$
$0.4, 0.424, \ 0.444, \ 0.488,$	$0.36, \ 0.355, \ 0.333, \ 0.323$	$0.625, \ 0.667, \ 0.625, \ 0.7$	$0.75, \ 0.77, \ 0.8, \ 0.875$
$0.4, 42.4\%, \frac{4}{9}, \ 0.488$	$36\%, 35.5\%, \frac{1}{3}, 0.323$	$\frac{5}{8}, 65.56\%, \frac{2}{3}, 0.7$	$\frac{3}{4}, 77\%, 0.8, \frac{7}{8}$

2. Distance & Time

2.1 Time Intervals

2.1 (a) I can calculate time intervals using both 12 and 24-hour time.

You should be able to calculate **time intervals** using both 12 or 24-hour time.

Aidan is going to drive from Invergordon to Edinburgh. He plans to leave Invergordon at 0925 and the drive will take him 3 hours and 25 minutes.

If Aidan plans to stop just once at Mcdonalds in Perth, for 30 minutes, at what time will he arrive in Edinburgh?

$0925 \xrightarrow{+3\ hours\ 25\ mins} 1250$

$\xrightarrow{+30\ mins} 1320$

Craig is moving house. He has booked a van to move his things. He wants to have all his things in his new house by 5.30 pm.

The journey will take 4 hours and 20 minutes while packing and unpacking the van will take 45 minutes each. What is the latest time he should leave?

$5.30\ pm \xrightarrow{-4\ hours\ 20\ mins} 1.10\ pm$

$\xrightarrow{-45\ mins} 12.25\ pm$

$\xrightarrow{-45\ mins} 11.40\ am$

Dan is travelling back from London to Glasgow. He leaves London at 08.20 am and stops for 15 minutes for a snack.

Dan then continues on his journey before stopping for lunch for 50 minutes. Dan arrives in Glasgow at 6.05 pm. How long was he driving for?

$6.05\ pm \xrightarrow{-15\ mins} 5.50\ pm$

$\xrightarrow{-50\ mins} 5\ pm$

$From\ 8.20\ am\ to\ 5\ pm\ there\ is\ 8\ hours\ 40\ mins.$

2.1 (b) I can work with time intervals and different time zones.

There are **24 time zones** around the world. The world's time is set from Greenwich in London and this central time zone is called **Greenwich Mean Time** (GMT).

Countries to the east of London are ahead of GMT while countries to the west of London are behind GMT.

Euan flies from Glasgow to Athens. The plane leaves Glasgow at 1940 and the flight lasts 2 hours 40 minutes. Athens time is 2 hours ahead of Edinburgh.

It should take 55 minutes to clear security. At what time will Euan leave the airport?

$1940 \xrightarrow{+2\ hours\ 40\ mins} 2220$

$\xrightarrow{+2\ hours\ (time\ zones)} 0000$

$\xrightarrow{+55\ mins} 0055$

James is on holiday in Chicago. His flight to Edinburgh departs Chicago at 0930 local time. The flight time is 7 hours 15 minutes.

If Chicago time is 6 hours behind Edinburgh time then calculate the local time when the plane lands in Edinburgh.

$0930 \xrightarrow{+7\ hours\ 15\ mins} 1645$

$\xrightarrow{+6\ hours\ (time\ zones)} 2245$

Eilidh flew from Sydney, Australia, to Manchester on Wednesday 17th February. The plane left Sydney at 2.20 pm. The total journey time, including a stopover in Dubai is 21 hours 15 minutes. Sydney time is 10 hours ahead of Manchester.

When did the plane arrive?

$Wed\ 17/02\ 2.20\ pm$

$\xrightarrow{+21\ hours\ 15\ mins}$

$Thu\ 18/02\ 11.05\ am$

$\xrightarrow{-10\ hours\ (time\ zones)}$

$Thu\ 18/02\ 1.05\ am$

2.2 Distance & Speed

2.2 (a) I can convert between different units of time.

To convert **hours to minutes** you **multiply by 60**. To convert **minutes to hours** you **divide by 60**.

Hours	0.7 hrs	$0.9\ hrs$	1.3 hrs	$1.1\ hrs$	5.75 hrs	$3.25\ hrs$
Hours & Mins	$42\ mins$	54 mins	$1\ hr\ 18\ mins$	1 hr 6 mins	$5\ hrs\ 45\ mins$	3 hrs 15 mins

As well as converting between hours and minutes you should also be able to convert between other units of time.

Seconds → Divide by 60 → ← Multiply by 60 ← **Minutes** → Divide by 60 → ← Multiply by 60 ← **Hours** → Divide by 24 → ← Multiply by 24 ← **Days**

Change 4 030 s to hours, minutes and seconds. $67\ mins\ 10\ s$ $= 1\ hours\ 7\ mins\ 10\ s$	Change 2 000 minutes to days, hours and minutes. $33\ hours\ 20\ mins$ $= 1\ day\ 7\ hours\ 10\ mins$	Change 9 100 s to hours, minutes and seconds. $151\ mins\ 40\ s$ $= 2\ hours\ 31\ mins\ 40\ s$	Change 2 825 minutes to days, hours and minutes. $47\ hours\ 5\ mins$ $= 1\ day\ 23\ hours\ 5\ mins$

2.2 (b) I can calculate distance, speed and time.

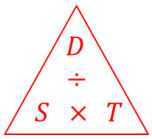
You should be able to calculate **distance**, **speed** and **time**.

$D = S \times T$ $S = D \div T$ $T = D \div S$

Lewis completes a lap of a 1.021 km go-kart track in 1 minute 42.3 seconds.

What was Lewis' average speed in his lap. Round your answer to the nearest km/h.

$T = 102.3\ s$
$= 0.028\ hours$
$S = 1.021 \div 0.028$
$= 36.46$
$= 36\ km/h$



Kyle arrived at a meeting at 11:20. He travelled 225 miles to his meeting at an average speed of 60 mph. During his journey he stopped for half an hour.

When did he leave home?

$T = 225 \div 60$
$= 3.75\ hours$
$= 3\ hours\ 45\ mins$



$11{:}20 \xrightarrow{-3\ hours\ 45\ mins} 07{:}35$
$\xrightarrow{-30\ mins} 07{:}05$

Samantha leaves home at 8.40 am and arrives at work at 8.55 am. Her average speed was 7.2 m/s.

What distance did Samantha travel? Give your answer in kilometers.

$From\ 8.40\ am\ to\ 8.55\ am\ there\ is\ 15\ mins.$
$T = 900\ s$
$D = 7.2 \times 900$
$= 6\ 480\ m$
$= 6.48\ km$



3. Tables & Scale

3.1 Reading Tables

3.1 (a) I can read and interpret information displayed in a table.

You should be able to read and interpret information displayed in a **table**.

The table shows the monthly life insurance premiums per £1 000.

Males Aged	16 - 24	25
Female Aged	16 - 30	31
Non-smoker	1.80	1.90
Smoker	2.30	2.40

James and Katie both want insurance for £20 000. Calculate how much more James, 25, and a smoker would pay as a monthly premium than Katie, 23, and a non-smoker.

$James: 20 \times 2.40 = £48$

$Katie: 20 \times 1.80 = £36$

James would pay £12 *more.*

The table shows different rates of travel insurance for adults and children in Europe and Worldwide.

D	Europe		Worldwide	
	A	C	A	C
5	6.04	4.15	32.08	16.04
10	8.12	6.44	44.80	22.40

Mr and Mrs Irving are planning to go for a week long holiday to New York with their three young daughters. They have set aside £150 for insurance. Is this enough?

$2 \times 44.80 + 3 \times 22.40 = £156.80$

$156.80 - 150 = £6.80$

No are £6.80 *short.*

Tain - Dingwall - Inverness			
Tain	1320	1350	1420
Alness	1342	1412	1442
Evanton	1350	1420	1450
Dingwall	1410	1430	1500
Inverness	1435	1455	1525
Aviemore	1525	1545	1615

Dan and Steph are staying in Alness. They need to get to Inverness by 1500. It takes 15 minutes to walk to the bus station. What is the latest time that they should leave home?

$1412 \xrightarrow{-15\ mins} 1357$

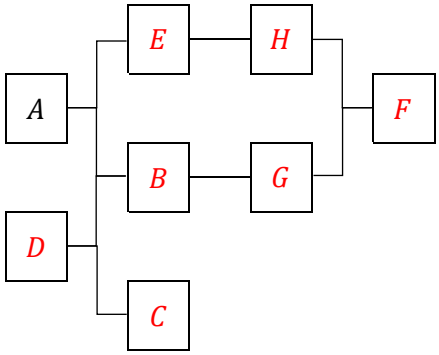
The latest they should leave is 1357.

3.2 Precedence Tables

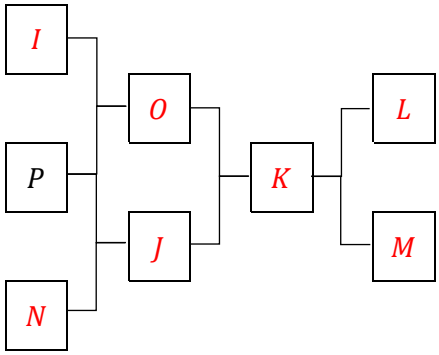
3.2 (a) I can read and construct precedence tables.

A **precedence table** is a tool for scheduling activities in planning a project. It is based on the logic of what activities must follow other activities and what activities can run at the same time.

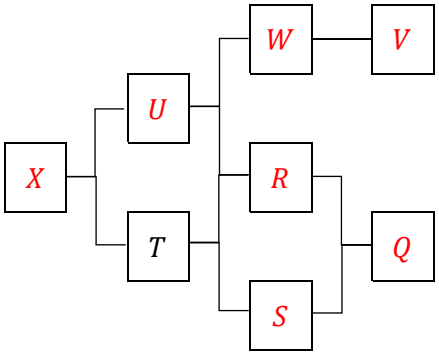
Tasks	Preceding Tasks
A	None
B	A
C	D
D	None
E	A
F	G, H
G	B
H	E



Tasks	Preceding Tasks
I	None
J	N, P
K	O, J
L	K
M	K
N	None
O	I, P
P	None



Tasks	Preceding Tasks
Q	R, S
R	U, T
S	T
T	X
U	X
V	W
W	U
X	None

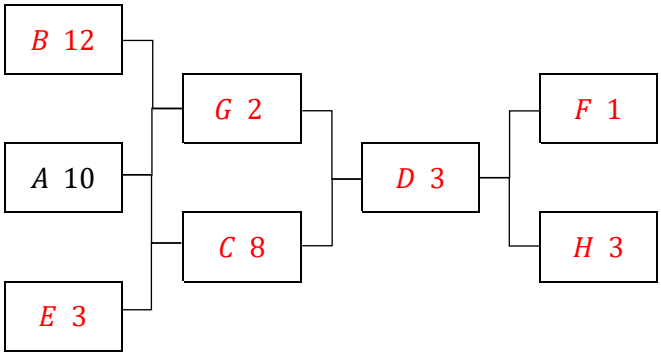


3.2 (b) I can make decisions based on precedence tables.

Questions involving precedence tables will often involve you making a decision based on timings.

A TV company is thinking about producing a new kids cartoon. The following table of necessary tasks was produced.

Activity	Description	Preceding Task	Time (months)
A	Create Script	None	10
B	Animation	None	12
C	Voice Work	A, E	8
D	Screen Test	C, G	3
E	Find Cast	None	3
F	Post Production	D	1
G	Editing	A, B	2
H	Advertise	D	3



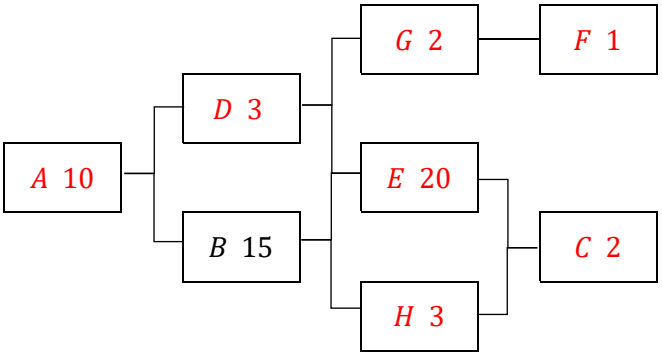
By filling in the diagram above decide will the cartoon be ready within 2 years?

$12 + 8 + 3 + 3 = 26\ months$

$No,\ 2\ months\ more\ are\ required.$

A family are looking to add an extension to their house. The following table of necessary tasks was produced.

Activity	Description	Preceding Task	Time (weeks)
A	Create Plan	None	10
B	Planning Permit	A	15
C	Arrange Furniture	E, H	2
D	Cost Material	A	3
E	Building Work	B, D	20
F	Pay Costs	G	1
G	Arrange Loan	D	2
H	Buy Furniture	B	3



By filling in the diagram above decide will the extension be completed within 48 weeks?

$10 + 15 + 20 + 2 = 47\ weeks$

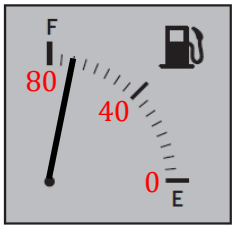
$Yes,\ 1\ month\ to\ spare.$

3.3 Tables & Scale

3.3 (a) I can read values from a scale.

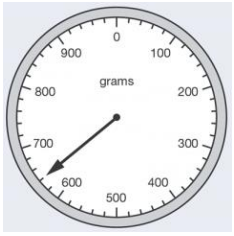
You should be able read values from a **scale.**

The tank holds 80 litres of fuel when full. How much is currently in it?



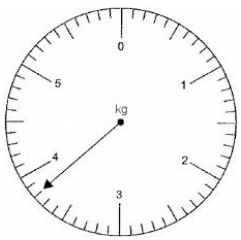
$70\ litres$

What weight is shown on the scales below. Give your answer in kg.



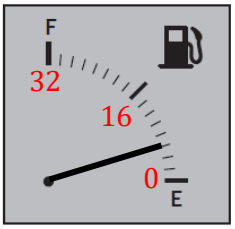
$0.64\ kilograms$

What weight is shown on the scale below? Give your answer in kg.



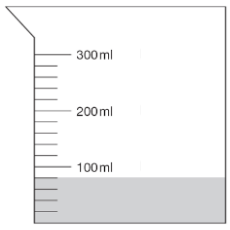
$3.8\ kilograms$

The tank holds 32 litres of fuel when full. How much is currently in it?



$6\ litres$

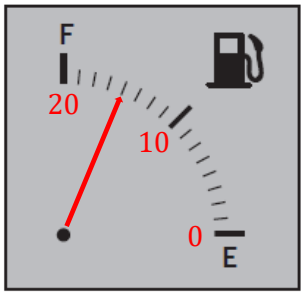
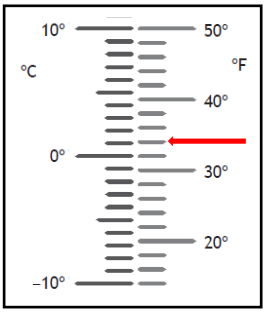
What measurement is shown on the measuring jug below.



$80\ ml$

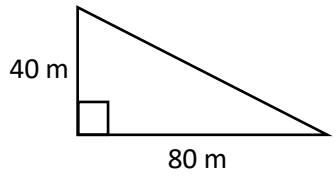
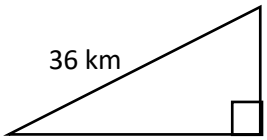
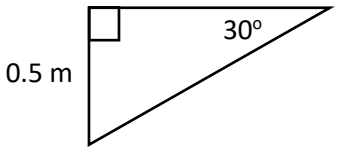
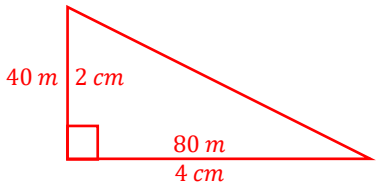
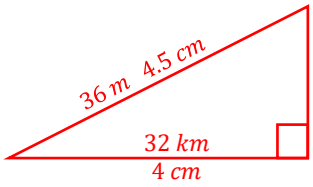
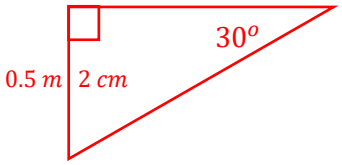
3.3 (b) I can mark values down on a scale.

You should be able to accurately mark down values on a scale.

Mark -14°C on the thermometer below.	Mark 2.4 kilograms on the scale below.	The tank holds 20 litres. Mark 15 litres.	Mark 34°F on the thermometer below.
			

3.3 (c) I can make a scale drawing using a protractor.

You should also be able to create **scale drawings** using a **ruler** and a **protractor**.

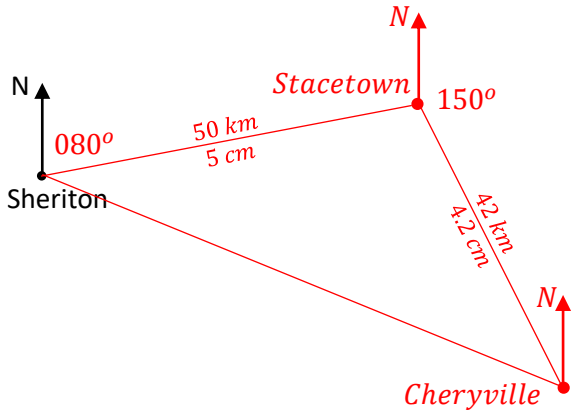
Construct a scale drawing of the following using a scale 1 cm : 20 m.	Construct a scale drawing of the following using a scale 1 cm : 8 km.	Construct a scale drawing of the following using scale 1 cm : 0.25 m.
		
$80 \div 20 = 4\ cm$ $40 \div 20 = 2\ cm$	$32 \div 8 = 4\ cm$ $36 \div 8 = 4.5\ cm$	$0.5 \div 0.25 = 2\ cm$
		
Find the size of the missing angles. $27^o, 63^o\ (using\ a\ protractor)$	Find the size of the missing side. $2.1\ cm\ (using\ a\ ruler) = 8.4\ cm$	Find the size of the missing sides. $3.5\ cm\ (using\ a\ ruler) = 0.875\ m$ $4\ cm\ (using\ a\ ruler) = 1\ m$

3.3 (d) I can make a scale drawing using bearings.

Bearings are used to describe the direction in which to travel to get from one place to another.

- They are measured from the **north** line in a **clockwise direction**.
- A bearing can be any angle from 0° to 360° and is written as a **three-figure number**.

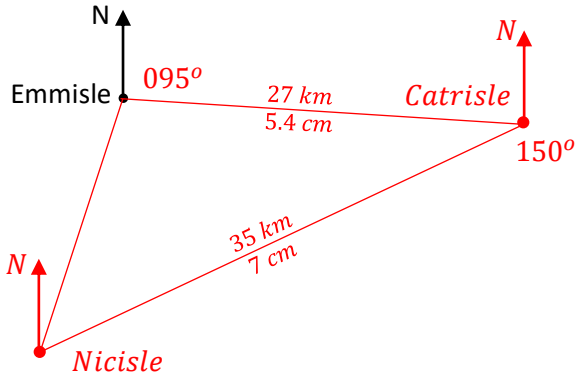
A helicopter flew from Sheriton to Stacetown and then onto Cheryville. It flew 50 km on a bearing of 080° to Stacetown. It then flew 42 km on a bearing of 150° to Cheryville. Construct a scale drawing to illustrate this journey. Use a scale of 1 cm : 10 km.	A boat sailed from Emmisle to Catrisle and then onto Nicisle. It sailed 27 km on a bearing of 094° to Catrisle. It then sailed 35 km on a bearing of 243° to Nicisle. Construct a scale drawing to illustrate this journey. Use a scale of 1 cm : 5 km.



The helicopter then returns to Sheriton from Cheryville. Find the distance and bearing of this flight.

$Distance = 7.5\ cm\ (using\ a\ ruler) = 7.5\ km$

$Bearing = 292^o\ (using\ a\ protractor)$



The boat then returns to Emmisle from Nicisle. Find the distance and bearing of this journey.

$Distance = 3.5\ cm\ (using\ a\ ruler) = 17.5\ km$

$Bearing = 341^o\ (using\ a\ protractor)$

4. Money Problems

4.1 Appreciation

4.1 (a) I can carry out calculations involving appreciation and depreciation.

Appreciation means an **increase** in size. **Depreciation** means a **decrease** in size.

You can calculate a **percentage increase** or **decrease** in one simple step using a **multiplier.**

	Increase						Decrease					
Percentage	2%	5%	10%	16%	30%	1.4%	4%	9%	10%	18%	40%	2.7%
Multiplier	1.02	1.05	1.1	1.16	1.3	1.014	0.96	0.91	0.9	0.82	0.6	0.973

You can solve problems on **appreciation** and **depreciation** using **multipliers** and **powers**.

Norman puts £650 in a savings account which offers 2.1% interest annually.

What will this be worth in 7 years?

$650 \times 1.021^7 = £751.78$

A car is bought for £900. It's value decreases by 6.2% a year.

What will its value be in 4 years to 3 significant figures?

$900 \times 0.938^4 = 696.71$

$= £697$

The population of a town, Alness, is growing by 5% annually.

If the population is currently 6 000 what will it be in 2 years.

$6\,000 \times 1.05^2 = 6\,615$

Ewan puts £400 in a savings account which offers 3.7% interest annually.

What will this be worth in 3 years?

$400 \times 1.037^3 = £446.06$

4.1 (b) I can work with more challenging appreciation questions.

You may be given a question on **appreciation** or **depreciation** requiring some more thought.

A bus is worth £20 000 it will be sold when its value is less than £15 000. If its value depreciates by 10% a year, when will it be sold?

$1: 20\,000 \times 0.9^1$

$= £18\,000$

$2: 20\,000 \times 0.9^2$

$= £16\,200$

$3: 20\,000 \times 0.9^3$

$= £14\,580$

It will be sold after 3 years.

The population of a city is rising by 20% a year. If its population is 200 000 after how many years will its population exceed 400 000?

$1: 200\,000 \times 1.2^1$

$= 240\,000$

$2: 200\,000 \times 1.2^2$

$= 288\,000$

$3: 200\,000 \times 1.2^3$

$= 345\,600$

$4: 200\,000 \times 1.2^4$

$= 414\,720$

It will exceed this after 4 years.

Lesley puts £12 500 in the bank. A year later it is worth £12 750. Calculate the percentage growth over the year.

$12\,750 - 12\,500 = 250$

$250 \div 12\,500 = 0.02$

$2\%\ growth$

If this rate continues for another 3 years what will it be worth?

$12\,750 \times 1.02^3$

$= £13\,530.40$

Katie puts £24 000 in the bank. A year later it is worth £24 744. Calculate the percentage growth over the year.

$24\,744 - 24\,000 = 744$

$744 \div 24\,000 = 0.031$

$3.1\%\ growth$

If this rate continues for another 5 years what will it be worth?

$24\,744 \times 1.031^5$

$= £28\,824.60$

4.1 (c) I can work with questions involving multiple percentage changes.

You should also be able to work with questions involving **multiple percentage changes**.

Erin bought a sports car for £15 000. Its value decreased by 3% the first year and then increased by 1.2% annually for the next 3 years. What was its value, to 3 significant figures at the end of this? $15\,000 \times 0.97 \times 1.012^3$ $= 15\,080.11$ $= £15\,100$	Abbie put £750 in her bank account. The value increased by 3.2% a year for the first 2 years and then by 0.9% a year for the next 3 years. How much was in the account to 2 significant figures? $750 \times 1.032^2 \times 1.009^3$ $= 820.53$ $= £820$	Chloe bought a trailer for £720. Its value decreased by 5% the first three years and then increased by 2.4% the fourth year. What was its value, to 2 significant figures at the end of this period? $720 \times 0.95^3 \times 1.024$ $= 632.13$ $= £630$	Rachel put £1 000 in her bank account. The value of increased by 2.4% a year for the first 3 years and then fell by 1% a year for the next 2 years. How much was in the account to 3 significant figures? $1\,000 \times 1.024^3 \times 0.99^2$ $= 1\,052.37$ $= £1\,050$

4.2 Shares

4.2 (a) I can calculate profit and loss.

You should be able to calculate both **profit** and **loss**.

A bake sale has sales worth £90. The charity must pay 35% of this figure to the organisers. The ingredients for the cake cost £15. Find the net amount raised. $10\%\ of\ 90 = 9$ $25\%\ of\ 90 = 22.5$ $35\%\ of\ 90 = 9 + 22.5 = 31.5$ $90 - 31.5 - 15 = 58.5 - 15$ $= £43.50\ profit$	Mike bought twelve scooters for a total of £2 400. He sold four of the scooters at a 30% profit and the other eight in a sale at a 20% loss. Find the net loss. $2\,400 \div 3 = 800$ $10\%\ of\ 800 = 80$ $30\%\ of\ 800 = 3 \times 80 = 240$ $2\,400 - 800 = 1\,600$ $10\%\ of\ 1\,600 = 160$ $20\%\ of\ 1\,600 = 2 \times 160 = 320$ $320 - 240 = £80\ loss$	In 2017 MathsCo made a profit of £12 000. In 2018 MathsCo's profit decreased by 15% and in 2019 MathsCo made a loss of £6 500. Find the net profit. $10\%\ of\ 12\,000 = 1\,200$ $5\%\ of\ 12\,000 = 1\,200 \div 2 = 600$ $15\%\ of\ 12\,000 = 1\,200 + 600$ $= 1\,800$ $12\,000 - 1\,800 = 10\,200$ $12\,000 + 10\,200 - 6\,500$ $= 22\,200 - 6\,500$ $= £15\,700\ profit$

4.2 (b) I can perform calculations involving buying and selling shares.

Shares are the units of the ownership of a company. They are also known as **stocks** or **equities**. If you buy shares in a company, you hope to profit when the shares you own increase in value and can be sold for **profit.**

Val buys 400 shares for £900. When she sells them, the share price has dropped to £2.20. She has to pay a fee of 2.5% of her selling price. Calculate the loss she made. $400 \times 2.20 \times 0.975 = £858$ $900 - 858 = £42\ loss$	Naomi bought 255 shares for £3.50 per share. She later sold them for £4.60 per share. She had to pay commission of 4.1% of the total selling price. Calculate her profit. $255 \times 3.50 = £892.50$ $255 \times 4.60 \times 0.959 = £1\,124.91$ $1\,124.91 - 892.50$ $= £232.41\ profit$	Ashleigh owns 6 000 shares in a company worth a total of £2 364. She sells 5 000 of her shares. She was charged £11.60 on her sale. How much did she receive? $2\,364 \div 6\,000 \times 5\,000 = £1\,970$ $1\,970 - 11.60 = £1\,958.40$

4.3 Foreign Exchange

4.3 (a) I can convert between different currencies.

To convert pounds (£) into a foreign currency, **multiply** by the **exchange rate**.

To convert a foreign currency back to pounds, **divide** by the **exchange rate.**

If the exchange rate is £1 = 1.14 Euros how many Euros will you get for £60? $60 \times 1.14 = 68.40\ euros$	If the exchange rate is £1 = 1.62 US Dollars how many Dollars will you get for £200? $200 \times 1.62 = \$324$	If the exchange rate is £1 = 11.2 Kroner how many Pounds will you get for 444 Kroner? $444 \div 11.2 = £39.64$	If the exchange rate is £1 = 92.35 Indian Rupees how many Pounds will you get for 1500 Rupees? $1\,500 \div 92.35 = £16.24$

4.3 (b) I can convert between multiple different currencies.

Sometimes you may be given a question which requires you to work with multiple different currencies.

Rates of Exchange	
Pounds Sterling (£)	Other Currencies
1	1.09 Euro
1	8.06 Danish Kroner
1	1.22 Bulgarian Lev

Andy is doing a multi-city trip to Copenhagen (Denmark). He changes £800 into Danish Kroner first. How much Danish Kroner does he receive? $800 \times 8.06 = 6\,448\ DK$ Andy spends 1 209 Kroner whilst in Copenhagen. He changes the remaining Danish Kroner into Bulgarian Lev. How much Bulgarian Lev does he receive? $6\,448 - 1\,209 = 5\,239\ D.K.$ $5\,239 \div 8.06 = £650$ $650 \times 1.22 = 793\ B.L.$	Fiona is going on a holiday to Paris and then after a week onto Sofia, Bulgaria. She changes £1 200 into Euros. How many Euros will she receive? $1\,200 \times 1.09 = 1\,308\ Euros$ Fiona spends 75 Euros each day while in Paris. She changes her remaining Euros into Bulgarian Lev. How many Bulgarian Lev will she receive? $1\,308 - 75 \times 7 = 783\ Euros$ $783 \div 1.09 = £718.35$ $718.35 \times 1.22 = 876.39\ B.L.$	Monika is going to a concert in Spain from her home in Inverness. Her Mum orders the tickets costing 70.85 Euros. Her Mum lives in Denmark so she must pay her back in Danish Kroner. How many Danish Kroner must Monika give her Mum? $70.85 \div 1.09 = £65$ $65 \times 8.06 = 523.90\ D.K.$

4.3 (c) I can find the foreign exchange rate of two currencies.

If you know how many pounds you get for another currency, then you will be able to find the **exchange rate**.

If £300 can be exchanged for $411 what is the exchange rate? $411 \div 300 = 1.37$ $£1 = \$1.37$	If £400 can be exchanged for 464 Euros what is the exchange rate? $464 \div 400 = 1.16$ $£1 = 1.16\ Euros$	1 760 Turkish Lira can be exchanged for £220. Find the exchange rate. $1\,760 \div 220 = 8.00$ $£1 = 8.00\ T.L.$	6 948.20 Chilean Peso can be exchanged for £7. Find the exchange rate. $6\,948.20 \div 7 = 992.60$ $£1 = 992.60\ C.P.$

5. Proportion & Probability

5.1 Ratio Calculations

5.1 (a) I can share a total in a given ratio.

You should be able to share a total in a given **ratio**.

Ali, Kate and Jim are paid to deliver leaflets. They shared the £240 in the ratio of 3 : 5 : 7 How much will Jim get? $3 + 5 + 7 = 15$ $240 \div 15 \times 7 = £112$	Fred, Gaby and Jo made money investing. They split the £15 300 profit in the ratio 5 : 3 : 1. How much will Fred get? $5 + 3 + 1 = 9$ $15\,300 \div 9 \times 5 = £8\,500$	There are 72 sweets which are blue, green and red in the ratio of 3 : 4 : 1. How many green sweets are there? $3 + 4 + 1 = 8$ $72 \div 8 \times 4 = 36\ sweets$	A £21 700 inheritance is to be split between Irene, Morag and Sarah in the ratio of 2 : 3 : 2. What will Morag get? $2 + 3 + 2 = 7$ $21\,700 \div 7 \times 3 = £9\,300$

5.1 (b) I can use a given ratio to solve a problem.

You should also be able to solve problems involving ratio.

An estate was shared between three brothers; Robert, John and Kieran, in the ratio of 2 : 3 : 5. If Robert received £1 600, how much will John get? $1\,600 \div 2 \times 3 = £2\,400$	Karen and Tim split a lottery win in the ratio of 3 : 4. Tim got £2 000. Karen claimed she should get £1 600. Is Karen correct? $2\,000 \div 4 \times 3 = £1\,500$ *Karen should only receive* £1 500.	As estate was shared between Andy, Suzy and Jenny, in the ratio 1 : 4 : 7. Suzy was given £500. Jenny claimed she should get £945. Is she correct? $500 \div 4 \times 7 = £875$ *Jenny should only receive* £875.	Claire, Emma and Nicola are paid to deliver leaflets. They shared the pay in a ratio of 5 : 6 : 9. Nicola gets £135. How much was their pay? $5 + 6 + 9 = 20$ $135 \div 9 \times 20 = £300$

5.2 Direct Proportion

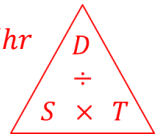
5.2 (a) I can use direct proportion to solve a problem.

Two quantities are in **direct proportion** when they increase or decrease in the **same ratio**.

A supermarket sells Irn Bru in 300 millilitre cans. One individual can costs just 72 pence. Find how much 1 litre of Irn Bru would cost? $72 \div 300 \times 1\,000$ $= 240\ p = £2.40$	Linda 12, Elaine, 11, Rory, 7 and David 4 are given money in proportion to their ages. If Linda is given £900 then how much will Rory get? $900 \div 12 \times 7 = £525$	Craig spends £2, Al £1.50 and Edith 50 p on lottery tickets. They split the win in proportion to this. Al receives £126 540. How much was their win? $2 + 1.50 + 0.50 = 4$ $126\,540 \div 1.50 \times 4$ $= £337\,440$	Hugh 10, Liz, 8, Bill, 6 and Roddy, 4 are given money in proportion to their ages. Roddy is given £600. How much will Hugh get? $600 \div 4 \times 10 = £1\,500$

5.2 (b) I can convert between different types of units.

You should be able to use **direct proportion** to convert between different types of **units**.

Jane's car is full of petrol. She uses 45% of the fuel on her journey to her holiday home. If her tank holds 53 gallons how much will she have left in the tank in litres? 1 gallon = 4.545 litres. 53×4.545 $= 240.885\ litres$ 240.885×0.45 $= 108.398\ litres$ $240.885 - 108.398$ $= 132.487\ litres$	Danielle drives 110 kilometres in 2 hours and 18 minutes. Calculate her average speed in miles per hour for this whole journey to 2 significant figures. 1 mile = 1.609 km. $D = 110 \div 1.609$ $= 68.37\ miles$ $T = 2.3\ hours$ $S = 68.37 \div 2.3$ $= 29.73$ $= 30\ mph$ 	Elicia wants to drive 960 km. Her car covers 45 miles per gallon of fuel and her fuel tank holds 62 litres of fuel when full. Will Elicia complete her journey with one full tank of fuel? $45 \times 1.609 = 72.405\ km$ $62 \div 4.545$ $= 13.64\ gallons$ 72.405×13.64 $= 987.60\ km$ $Yes\ she\ will\ manage\ as$ $987.60 > 960\ km.$	Steve drives 240 miles in 4 hours and 12 minutes. Calculate his average speed in kilometres per hour for this whole journey to 1 significant figure. 1 kilometre = 0.621 km. $D = 240 \div 0.621$ $= 386.47\ km$ $T = 4.2\ hours$ $S = 386.47 \div 4.2$ $= 92.02$ $= 90\ km/hr$ 

5.3 Inverse Proportion

5.3 (a) I can use inverse proportion to solve a problem.

While **direct proportion** means when one quantity increases, the other also increases, **inverse proportion** means when one quantity increases, the other decreases. For example, the faster you run the less time it takes.

It takes 8 hours for three men to paint the internal walls in a new building. How long would it have taken if one more colleague was to join them in this work? $8 \times 3 \div 4 = 6\ hours$	A town of 144 people is under siege. It has enough food to last the population for 24 days. If they take in an extra 48 people, how long will the food supply now last? $144 \times 24 \div 192$ $= 18\ hours$	It takes 5 bakers 6 hours to decorate a wedding cake. Calculate the time taken for 4 bakers to decorate the cake. Give your answer in hours and minutes. $5 \times 6 \div 4 = 7.5\ hours$ $= 7\ hours\ 30\ mins$	There is enough oxygen in a cave for 6 people to last 9 hours. How long would it last with 2 more people in the cave? Give your answer in hours and minutes. $5 \times 9 \div 8 = 6.75\ hours$ $= 6\ hours\ 45\ mins$

5.4 Probability

5.4 (a) I can calculate the probability of a single event happening.

Probability is about estimating how likely (probable) something is to happen.

A fair 6 sided dice is thrown. What is the probability that the number it lands on is greater than 4? $P(5,6) = \frac{2}{6} = \frac{1}{3}$	A letter is chosen at random from the word **BATHGATE**. What is the probability of selecting an **A**? $P(A) = \frac{2}{8} = \frac{1}{4}$	In a bag of sweets, 4 are red, 7 are blue and 3 are yellow. What is the probability of selecting a red? $P(red) = \frac{4}{14} = \frac{2}{7}$	A class is made up of 12 boys and 18 girls.What is the probability of selecting a girl from the class? $P(girl) = \frac{18}{30} = \frac{3}{5}$

5.4 (b) I can compare two probabilities as fractions or decimals.

You can make **decisions based on probability** by comparing them as either decimals or fractions.

There are two raffles.

Raffle A: 100 tickets, 6 prizes.

Raffle B: 60 tickets, 4 prizes.

Which raffle gives the better chance of a prize?

Use fractions.

$P(A) = \frac{6}{100} = \frac{3}{50} = \frac{9}{150}$

$P(B) = \frac{4}{60} = \frac{1}{15} = \frac{10}{150}$

Raffle B has a better chance as $\frac{10}{150} > \frac{9}{150}$.

Two classes sat a test.

Class 1: 23 out of 30 pupils passed.

Class 2: 15 out of 22 pupils passed.

Which class gave pupils a better chance of passing?

Use decimals.

$P(1) = \frac{23}{30} = 0.767$

$P(2) = \frac{15}{22} = 0.682$

Class 1 has a better pass rate as $0.767 > 0.682$.

There are two raffles.

Raffle A: 40 tickets, 5 prizes.

Raffle B: 60 tickets, 6 prizes.

Which raffle gives the better chance of a prize?

Use fractions.

$P(A) = \frac{5}{40} = \frac{1}{8}$

$P(B) = \frac{6}{60} = \frac{1}{10}$

Raffle A has a better chance as $\frac{1}{8} > \frac{1}{10}$.

There are two quizzes.

Quiz A: 40 people, 3 prizes.

Quiz B: 70 people, 5 prizes.

Which quiz gives the better chance of a prize?

Use decimals.

$P(A) = \frac{3}{40} = 0.075$

$P(B) = \frac{5}{70} = 0.071$

There is a better chance in Competition A as $0.075 > 0.071$.

5.4 (c) I can make decisions based on expected frequencies.

Probability can be used to make **predictions** based on **expected frequencies**.

Three out of five people shampoo for normal hair. If 100 people are interviewed, how many would you expect to use 'normal shampoo'?

$P(normal) = \frac{3}{5}$

$100 \div 5 \times 3 = 60$ *people*

Seven out of ten people drive to work. If 240 people are interviewed, how many would you expect to drive to work?

$P(drive) = \frac{7}{10}$

$240 \div 10 \times 7 = 168$ *people*

The probability of a box containing at least one broken bottle is 0.018. 15 boxes out of 900 have at least one broken bottle. Is this expected?

$0.018 \times 900 = 16.2$ *boxes*

You would expect more than 15 *boxes as* $16.2 > 15$.

5.4 (d) I can calculate the probability of combined events.

You should also be able to calculate the **probability** of **two independent events**.

Two dice are thrown and their numbers are multiplied together. What is the probability that you will get a total of 20 or more?

$6 \times 6 = 36$ *possibilities*

$P(20 \text{ or more}) = \frac{8}{36}$

$= \frac{2}{9}$

Dice 1	*Dice 2*
4	5, 6
5	4, 5, 6
6	4, 5, 6

Two spinners, one with numbers 1 - 3 and the other with numbers 1 - 7 are spun. What is the probability the numbers will add to less than 6?

$3 \times 7 = 21$ *possibilities*

$P(\text{less than } 6) = \frac{9}{21}$

$= \frac{3}{7}$

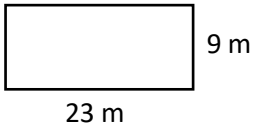
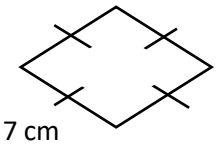
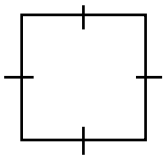
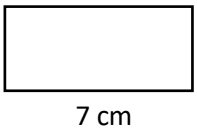
Spinner 1	*Spinner 2*
1	1, 2, 3, 4
2	1, 2, 3
3	1, 2

6. 2D Shapes

6.1 Perimeter

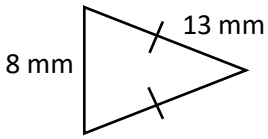
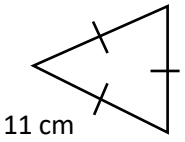
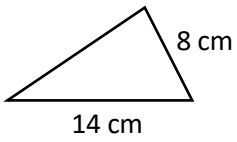
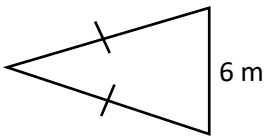
6.1 (a) I can calculate the perimeter of a quadrilateral.

You should be able to find the **perimeter** of a **quadrilateral**.

Find the perimeter of the shape below.  $23 + 9 + 23 + 9 = 64\ m$	Find the perimeter of the shape below.  $7 + 7 + 7 + 7 = 28\ cm$	The perimeter is 84 mm find the missing sides.  $84 \div 4 = 21\ mm\ each$	The perimeter is 20 cm find the missing side.  $20 - 7 - 7 = 6\ cm$ $6 \div 2 = 3\ cm\ each$

6.1 (b) I can calculate the perimeter of a triangle.

You should also be able to find the **perimeter** of a **triangle**.

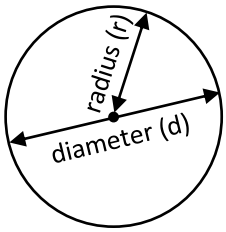
Find the perimeter of the shape below.  $13 + 8 + 13 = 34\ mm$	Find the perimeter of the shape below.  $11 + 11 + 11 = 33\ cm$	The perimeter is 34 cm find the missing side.  $34 - 14 - 8 = 12\ cm$	The perimeter is 26 m find the missing sides.  $26 - 6 = 20\ m$ $20 \div 2 = 10\ m\ each$

6.1 (c) I can calculate the circumference of a circle.

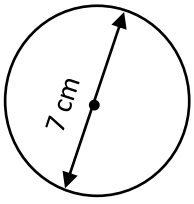
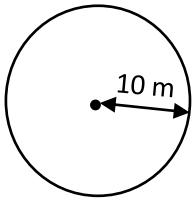
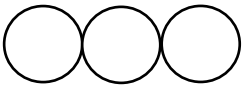
The distance around a shape is its **perimeter**. The **perimeter** of a circle is its **circumference**.

The **circumference** of a circle is given in the **formulae sheet**:

$$C = \pi d$$

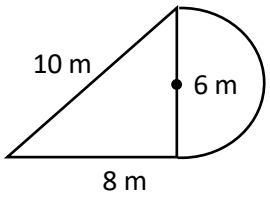
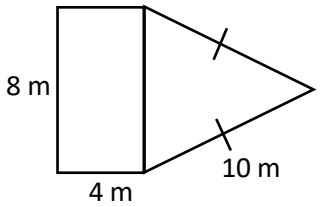
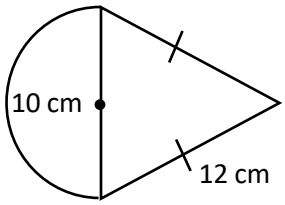
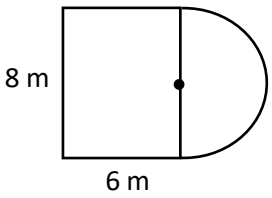


Note: The diameter and radius of a circle are linked as $d = r \times 2$ and $r = d \div 2$.

Calculate the perimeter of the circle.  $C = \pi \times 7$ $= 21.99\ cm$	Find the perimeter using π = 3.14.  $C = 3.14 \times 20$ $= 62.8\ m$	A decoration is made up of three identical circles.  If the decoration is 27 cm long what is the radius of each circle? $27 \div 6 = 4.5\ cm$	A car wheel has radius 26 cm. On a journey each wheel turns 2 460 times. How far does it travel? $C = \pi \times 52$ $= 163.36\ cm$ $2\,460 \times 163.36$ $= 401\,872.53\ cm$

6.1 (d) I can calculate the perimeter of a composite shape.

A **composite shape** is one that is made up of at least two **regular shapes.**

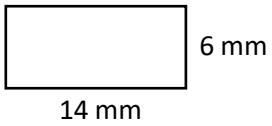
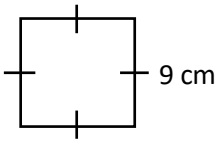
Find the perimeter of the following composite shape. Do not use a calculator for this and instead use the fact that π = 3.14.  $C_{semi-c} = 3.14 \times 6 \div 2$ $= 3.14 \times 3$ $= 9.42\ m$ $P_{total} = 9.42 + 10 + 8$ $= 27.42\ m$	The following garden needs a fence around the perimeter. Fencing can be bought in 10 metre sections costing £142. Find the total cost.  $P = 4 + 8 + 4 + 10 + 10$ $= 36\ m$ $36 \div 10 = 3.6$ $\approx 4\ lengths\ required$ $4 \times 142 = £568$	The following shape represents a broach. A gold thread will be put around this. If 1 cm of gold thread costs £3.12. Find the total cost.  $C_{semi-c} = \pi \times 10 \div 2$ $= 15.71\ m$ $P_{total} = 15.71 + 12 + 12$ $= 39.71\ cm$ $\approx 40\ cm\ required$ $40 \times 3.20 = £124.80$	Find the perimeter of the following composite shape. Do not use a calculator for this and instead use the fact that π = 3.14.  $C_{semi-c} = 3.14 \times 8 \div 2$ $= 3.14 \times 4$ $= 12.56\ m$ P_{total} $= 12.56 + 6 + 8 + 6$ $= 42.56\ m$

6.2 Area

6.2 (a) I can calculate the area of a rectangle.

You should know the area a **rectangle**.

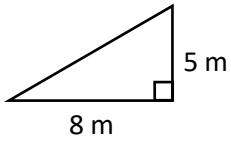
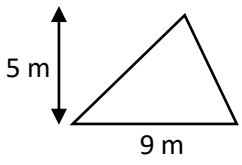
$$A_{rectangle} = L \times B$$

Calculate the area.  $A_{rectangle} = 14 \times 6$ $= 84\ mm^2$	Calculate the area.  $A_{rectangle} = 9 \times 9$ $= 81\ cm^2$	The area of a rectangle is 48 cm^2. If the dimension of one of its sides is 3 cm find the other dimension. $48 \div 3 = 16\ cm$	The area of a certain square is 121 mm^2. Find the length of the sides of this square. $\sqrt{121} = 11\ mm$

6.2 (b) I can calculate the area of a triangle.

You should know the area a **triangle**.

$$A_{triangle} = \frac{1}{2} \times b \times h$$

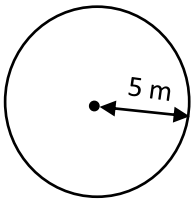
Calculate the area.  $A_{triangle} = \frac{1}{2} \times 8 \times 5$ $= 20\ m^2$	Calculate the area.  $A_{triangle} = \frac{1}{2} \times 9 \times 5$ $= 22.5\ m^2$	The area of a triangle is 12 cm^2. If the base of the triangle is 8 cm find the height. $12 \times 2 = 24$ $24 \div 8 = 3\ cm$	The area of a triangle is 30 m^2. If the height of the triangle is 8 cm find the base. $30 \times 2 = 60$ $60 \div 8 = 7.5\ m$

6.2 (c) I can calculate the area of a circle.

The **area** of a circle is given in the **formulae sheet**:

$$A_{circle} = \pi r^2$$

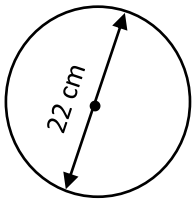
Calculate the area.



$A_{circle} = \pi \times 5^2$

$= 78.54\ m^2$

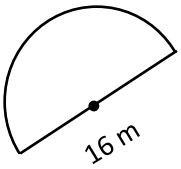
Calculate the area.



$A_{circle} = \pi \times 11^2$

$= 380.13\ cm^2$

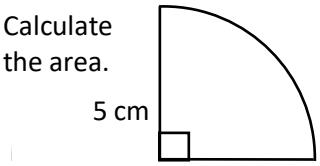
Calculate the area.



$A_{semi\ circle} = \pi \times 8^2 \div 2$

$= 100.53\ m^2$

Calculate the area.



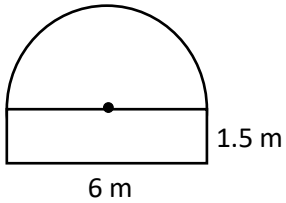
$A_{sector} = \pi \times 5^2 \div 4$

$= 19.63\ cm^2$

6.2 (d) I can calculate the area of a composite shape.

You should also be able to calculate the **area** of a **composite shape** by breaking it into parts.

The following composite shape is to be spray painted purple. Calculate the total area requiring to painted using π = 3.14.



$A_{rectangle} = 6 \times 1.5$

$= 9\ m^2$

$A_{semi-c} = 3.14 \times 3^2 \div 2$

$= 3.14 \times 9 \div 2$

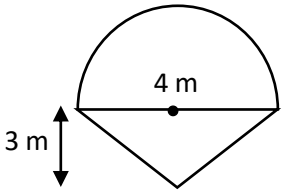
$= 28.26 \div 2$

$= 14.13\ m^2$

$A_{total} = 9 + 14.13$

$= 23.13\ m^2$

The following will be covered in gravel. A bag of gravel costs £24 and covers 4 m^2. Find the cost to cover this.



$A_{semi-c} = \pi \times 4^2 \div 2$

$= 25.13\ m^2$

$A_{triangle} = \frac{1}{2} \times 8 \times 3$

$= 12\ m^2$

$A_{total} = 25.13 + 12$

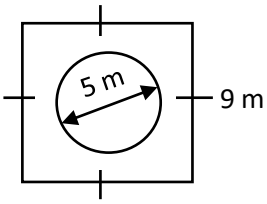
$= 37.13\ m^2$

$37.13 \div 4 = 9.28$

$\approx 10\ bags\ required$

$10 \times 24 = £240$

Moira has a small garden with a pond in the middle. Bags of seed cost £1.20 and cover an area of 7 m^2. How much will this cost?



$A_{rectangle} = 9 \times 9$

$= 81\ m^2$

$A_{circle} = \pi \times 2.5^2$

$= 19.63\ m^2$

$A_{total} = 81 - 19.63$

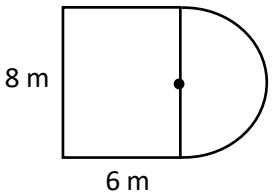
$= 61.37\ m^2$

$61.37 \div 7 = 8.77$

$\approx 9\ bags\ required$

$9 \times 1.20 = £10.80$

Find the area of the following shape. Do not use a calculator for this and instead use the fact that π = 3.14.



$A_{rectangle} = 8 \times 6$

$= 48\ m^2$

$A_{semi-c} = 3.14 \times 4^2 \div 2$

$= 3.14 \times 16 \div 2$

$= 3.14 \times 8$

$= 25.12\ m^2$

$A_{total} = 48 + 25.12$

$= 73.12\ m^2$

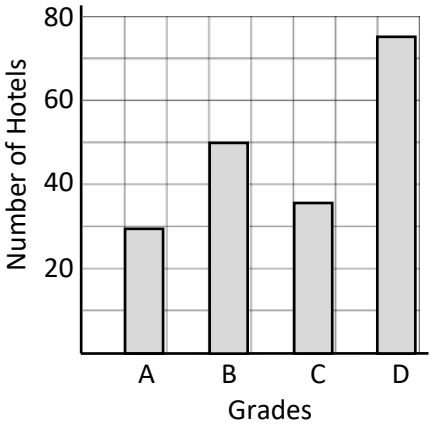
7. Graphs & Charts

7.1 Reading Diagrams

7.1 (a) I can read and interpret information displayed in a bar chart.

You should be able to read and interpret information displayed in a **bar chart**.

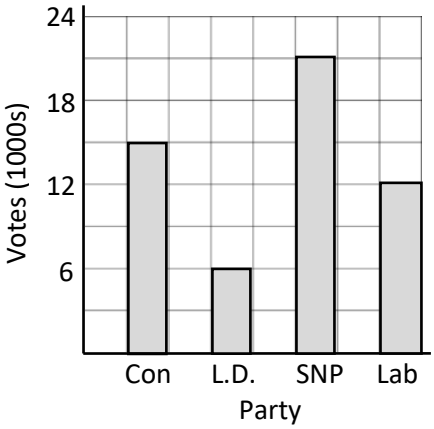
The graph shows the number of hotels awarded grades A to D.



What fraction of hotels were awarded a grade B?

$\frac{35}{190} = \frac{7}{38}$

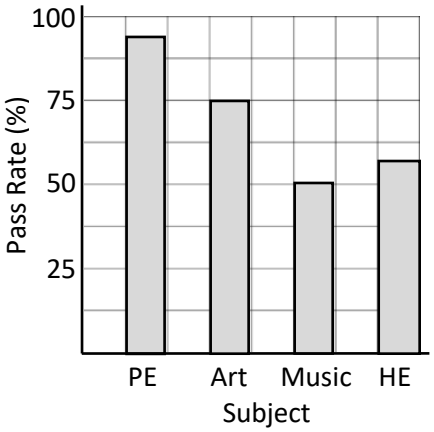
The results of the election are shown in the graph below.



What percentage of people voted for the Liberal Democrats?

$\frac{6}{54} = 11.1\%$

The graph shows the pass rates of different exams.



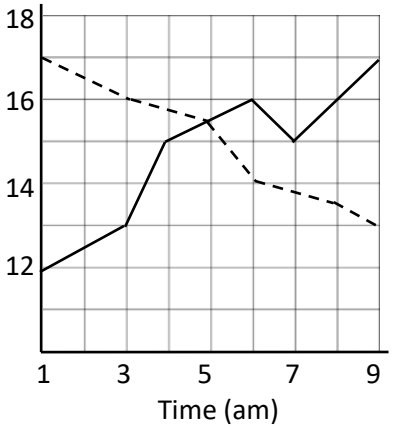
Estimate the percentage of pupils who passed HE.

60%

7.1 (b) I can read and interpret information displayed in a line graph.

You should also be able to read and interpret information displayed in a **line graph**.

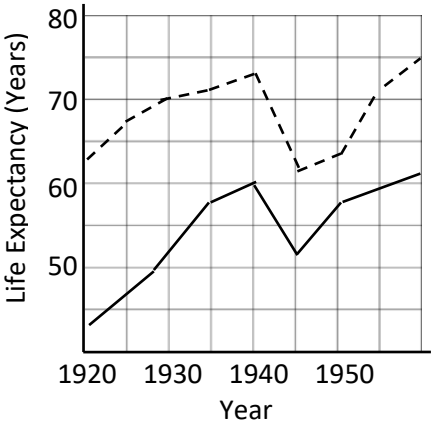
The temperature of two towns is measured each hour.



At what time did the two towns have the same temperature?

5 am

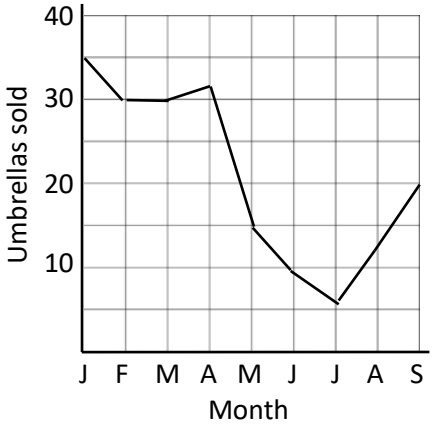
The life-expectancy of people in two countries is graphed.



Can you think why life expectancy fell at one point?

Possibly WW2.

The number of umbrellas sold per month in a shop is shown.



Why do you think the number of umbrellas sold fell?

Less rain in the summer.

7.1 (c) I can construct and interpret stem-and-leaf diagrams.

A **stem-and-leaf diagram** is used to present data, allowing you to see the distribution of numbers.

The diagram show a rugby club's players' ages.

2	3 5 5 8 8 9
3	0 3 3 3 7
4	2 5 8
5	4

2|3 means 23 years

What percentage are in their 30s?

$\frac{5}{15} = 33.33\%$

The diagram shows pupils' marks in a recent test.

5	2 6 8 8
6	0 4 4 4 5 9
7	2 3
8	1

5|2 means 52 marks

What fraction scored less than 64?

$\frac{5}{13}$

Construct a diagram to display the following:

73 80 85 77 81 70 63 68 78 77

6	3 8
7	0 3 7 7 8
8	0 1 5

$6 \mid 3 \; means \; 63$

7.1 (d) I can construct and interpret back to back stem-and-leaf diagrams.

A **back to back stem-and-leaf diagram** allows you to compare two sets of numbers.

The ages of two football teams are shown.

Team 1		Team 2
8 6 5 3 3 1	1	7 8 8
7 6 3 2 0	2	1 2 2 6 8
	3	2

3 | 2 means 32 years

Compare the ages of both teams.

Team 2 are younger on average.

The marks in a test for two classes are shown.

Class 1		Class 2
7 5 5 4 2	2	8 9
9 8 5 3	3	1 3 5 6 9
	4	2 4 7

2 | 2 means 22 marks

Compare the marks of both classes.

Class 2 have better marks in general.

Construct a stem-and-leaf diagram to show the marks for two classes:

Class 1: 23 31 20 17 38 22 19
Class 2: 11 14 21 19 24 27 10

Class 2		*Class 1*
9 4 1 0	1	7 9
7 4 1	2	0 2 3
	3	1 8

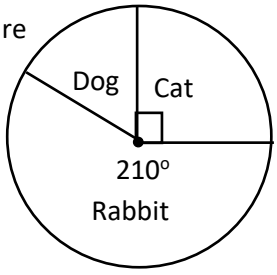
$1 \mid 7 \; means \; 17 \; marks$

7.2 Pie Charts

7.2 (a) I can interpret information displayed in a pie chart.

Pie charts are drawn as a **circle** representing all the data given. The 360° about the centre of the circle is divided up according to the proportions of the different quantities shown.

72 people were asked what pet they might buy next. How many chose a dog?

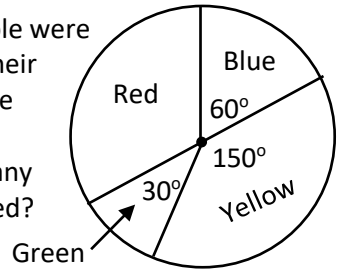


$360 - 210 - 90 = 60^o$

$\frac{60}{360} = \frac{1}{6}$

$72 \div 6 = 12$

60 people were asked their favourite colour. How many chose red?

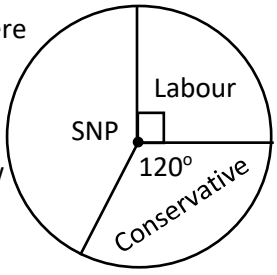


$360 - 60 - 150 - 30 = 120^o$

$\frac{120}{360} = \frac{1}{3}$

$60 \div 3 = 20$

48 people were asked which candidate they would vote for. How many chose the SNP?



$360 - 120 - 90 = 150^o$

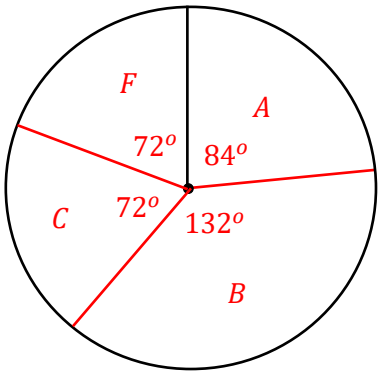
$\frac{150}{360} = \frac{5}{12}$

$48 \div 12 \times 5 = 20$

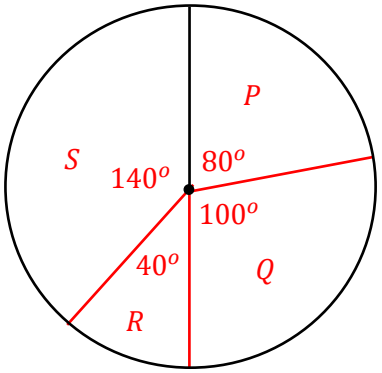
7.2 (b) I can construct a pie chart given the relevant information.

Pie charts can be drawn as long as you remember the crucial fact that there are 360° about the centre of the circle.

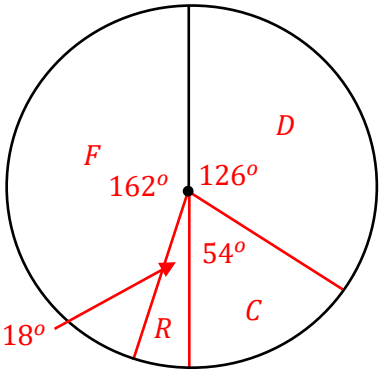
M	No	Angle at centre
A	7	$7 \div 30 \times 360 = 84^o$
B	11	$11 \div 30 \times 360 = 132^o$
C	6	$6 \div 30 \times 360 = 72^o$
F	6	$6 \div 30 \times 360 = 72^o$



C	No	Angle at centre
P	8	$8 \div 36 \times 360 = 80^o$
Q	10	$10 \div 36 \times 360 = 100^o$
R	4	$4 \div 36 \times 360 = 40^o$
S	14	$14 \div 36 \times 360 = 140^o$



P	No	Angle at centre
D	7	$7 \div 20 \times 360 = 126^o$
C	3	$3 \div 20 \times 360 = 54^o$
R	1	$1 \div 20 \times 360 = 18^o$
F	9	$9 \div 20 \times 360 = 162^o$



7.3 Scattergraphs

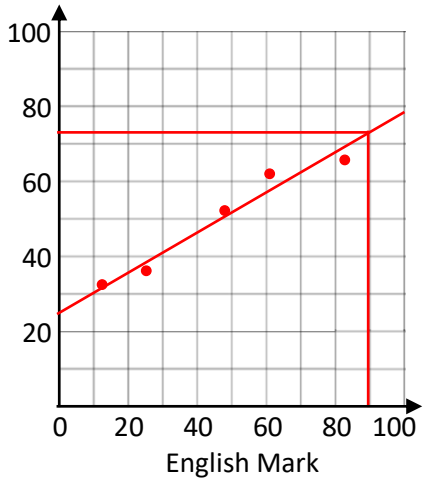
7.3 (a) I can plot and use a line of best fit.

Scattergraphs are used to represent and **compare two sets of data**. They are used to see whether there is any connection (**correlation**) between the two sets. If there is a **line of best fit** this can be plotted and then used.

The marks out of 100 in an English test and a History test are given.

Pupil	English	History
Jasmin	12	32
Logan	25	36
Jamie	48	52
Olivia	60	62
Billy	82	66

Plot these points and draw a line of best fit.



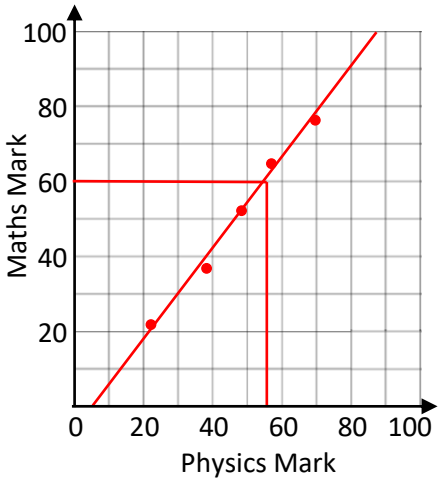
Holly scored 90 in her English test. Predict her History score.

73 marks

The marks out of 100 in a Maths test and a Physics test are given.

Pupil	Physics	Maths
Adam	22	22
Jack	39	37
Abbie	70	76
Levente	57	65
Nathan	48	52

Plot these points and draw a line of best fit.



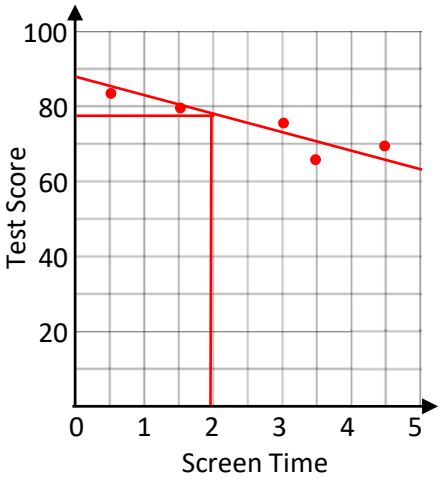
Megan scored 60 in her Maths test. Predict her Physics score.

57 marks

The average screen time and score in a test are given.

Person	Hours	Score
David	1.5	80
Kirsty	3	75
Euan	3.5	66
James	4.5	70
Eilidh	0.5	83

Plot these points and draw a line of best fit.



Hugh scored 78 marks in his test, estimate his screen time.

2 hours

8. Budgeting

8.1 Budgets

8.1 (a) I can carry out calculations involving a budget.

You should be able to carry out calculations based on a budget.

Ray has saved up £450 for his holiday in Hull. His travel there and back will cost £76.45. His hotel costs £42 a night and he will stay there 5 nights. How much spending money will he have? $450 - 76.45 - 5 \times 42$ $= £163.55$	Debbie has bought return tickets for £76.20 to her brothers house in London. She budgets that she will spend £45 each day she stays there. Deborah has £350, will this be enough for a week? $76.20 + 7 \times 45$ $= £391.20$ $391.20 - 350 = £41.20$ *No, she is £41.20 short.*	Robert has saved up £2 865. He needs to pay his Mum back £294.50. 20% of his remaining money will go to paying of his car loan. Will he have enough to pay a deposit of £1 900 for a new flat? $2\,865 - 294.50$ $= £2\,570.50$ $0.8 \times 2\,570.50$ $= £2\,056.40$ $2\,056.20 - 1\,900$ $= £156.20$ *Yes he has £156.20 to spare.*	Frank has bought return tickets for £120.40 to London. Frank finds that his hotel will cost £62.50 a night. Frank has saved up £400 will this be enough for him to go for a 4 day stay? $120.40 + 4 \times 62.50$ $= £370.40$ $400 - 370.40 = £29.60$ *Yes he has £29.60 to spare.*

8.1 (b) I can make decisions based on a budget.

You should be able to carry out calculations based on a budget.

Michael is saving £28 a week to buy a new bike costing £520, in time for his cycling holiday in 18 weeks. Will he have saved enough for the bike? $18 \times 28 = £504$ $520 - 504 = £16$ *No, he is £16 short.*	Dwight saves £7·50 every week for 12 weeks to buy a new watch costing £125. For how many more weeks will he need to save? $125 \div 7.50 = 16.67$ ≈ 17 *weeks* $17 - 12 = 5$ *weeks* *He will need a further 5 weeks.*	Andy saves £95 every month, hoping to buy a TV costing £775. The TV's price is reduced by 20%. How many months will he have to save for? $775 \times 0.8 = £620$ $620 \div 95 = 6.53$ ≈ 7 *months* *Andy will have to save for 7 months.*	Pam saves £25 a week for an outfit costing £210. Pam's partner Jim saves £32 per week for a £250 suit. Who will be able to buy their outfit first? $210 \div 25 = 8.4$ ≈ 9 *weeks* $250 \div 32 = 7.82$ ≈ 8 *weeks* *Jim will be able to buy his outfit first.*

8.2 Best Buys

8.2 (a) I can decide on the best deal to buy an item.

A common question you may be asked is to identify which deal is **cheaper** – the **best buy**.

Tickets to the zoo are shown: Adult's ticket - £15.40 Child's ticket - £8.60 Family Pass - £45 Should Mr and Mrs Sommerville and their children Andrew and Daisy buy a family pass? $2 \times 15.40 + 2 \times 8.60 = £48$ $48 - 45 = £3$ *They should buy the Family Pass as it would save £3.*	A tailor buys buttons in bulk. Online offers are shown. 1 000 from JDV - £15·50 2 500 from Yo Ho Ho - £38.00 4 000 from Button.com - £60·60 Which company is offering the best online deal? $JDV: 15.50 \div 1\,000 = £0.0155$ $YHH: 38.00 \div 2\,500 = £0.0152$ $BC: 60.60 \div 4\,000 = £0.01515$ *Button.com is the best as you pay the least per button.*	The following deals are available for large pizzas which cost £10.50: Deal 1 – Buy 3 get 1 free Deal 2 – 30% of when you buy 4 Deal 3 – Any 4 pizzas for £34 Which deal is best for buying 4 large pizzas? $D1: 3 \times 10.50 = £31.50$ $D2: 0.7 \times 4 \times 10.50 = £29.40$ $D3: £34$ *Deal 2 is the cheapest option of the three.*

8.3 Payment Plans

8.3 (a) I can calculate the cost of an item bought by hire purchase.

Buying goods by making an **initial deposit** and paying the remaining cost over a required number of weeks, months or years, is called **hire purchase**.

The disadvantage to paying by **hire purchase** is that the goods tend to be **more expensive**.

Rachel wants to buy a new TV worth £920. She pays a 20% deposit and then pays 8 monthly payments of £95.40. How much more does this cost? $0.2 \times 920 + 8 \times 95.40$ $= £947.20$ $947.20 - 920 = £27.20$	Joey bought a new carpet, costing £1 116.80 in total. He paid a deposit of £80 followed by 12 equal monthly instalments. How much was each instalment? $1\,116.80 - 80$ $= £1\,036.80$ $1\,036.80 \div 12 = £86.40$	A laptop costs £1 240. A payment plan costs 10% more than this. This plan requires a 20% deposit followed by 40 equal payments. What is each payment? $1\,240 \times 1.1 = £1\,364$ $1\,364 \times 0.8 = £1\,091.20$ $1\,091.20 \div 40 = £27.28$	Phoebe wants to buy a new phone worth £630. She pays a 15% deposit and then pays 7 monthly payments of £84.40. How much more does this cost? $0.15 \times 630 + 7 \times 84.40$ $= £685.30$ $685.30 - 630 = £55.30$

8.3 (b) I can work out how much is to be repaid after borrowing money.

Individuals and small to large companies sometimes need to borrow money. You should be able to calculate how much money should be repaid.

Daphne takes out a loan of £17 040. The interest plus the admin fee is 9.4% of the loan amount. The total amount will be paid back in 12 equal monthly payments. Calculate the monthly payment.	Niles takes out a loan of £6 400. The interest plus the admin fee is 6.2% of the loan amount. The total amount will be paid back in 5 equal monthly payments. Calculate the monthly payment.	Martin takes out a loan of £8 000. The interest plus the admin fee in 7.5% of the loan amount. The will be paid back in 8 equal payments. Find these payments. Do not use a calculator for this question.	Frasier takes out a loan of £3 600. The interest plus the admin fee is 8.2% of the loan amount. The total amount will be paid back in 8 equal monthly payments. Calculate the monthly payment.

$17\,040 \times 1.094$ $= £18\,641.76$ $18\,641.76 \div 12$ $= £1\,553.48$	$6\,400 \times 1.062$ $= £6\,796.80$ $6\,796.80 \div 5$ $= £1\,359.36$	$1\%\ of\ 800 = 8$ $7.5\%\ of\ 800 = 7.5 \times 8$ $= £60$ $800 + 60 = £860$ $860 \div 8 = £107.50$	$3\,600 \times 1.082$ $= £3\,895.20$ $3\,895.20 \div 8 = £486.90$

8.3 (c) I can calculate how much insurance will cost.

Paying a sum of money, to cover you in case an unfortunate event might happen, is called **insurance**.

Life insurance is usually referred to as **assurance**.

The table shows the monthly house and contents premiums per £10 000.

Group	Building	Contents
1	£1.70	£5.40
2	£2.50	£7.10
3	£3.10	£9.80

Ross lives in a category 2 house and he wants to insure the contents of his house for a total of £40 000. How much will his monthly premium be?

$4 \times 7.10 = £28.40$

The table shows the monthly life insurance premiums per £1 000.

Males Aged	16 - 25	26
Female Aged	16 - 30	31
Non-smoker	2.80	2.90
Smoker	3.30	3.55

David and Margaret both want insurance for £30 000. Calculate how much more Margaret, 28, and a smoker would pay as a monthly premium than David, 26, and a non-smoker.

$Margaret: 30 \times 3.30 = £99$

$David: 30 \times 2.90 = £87$

$Margaret\ would\ pay\ £12\ more.$

The table shows the monthly house and contents premiums per £10 000.

Group	Building	Contents
1	£1.70	£5.40
2	£2.50	£7.10
3	£3.10	£9.80

Ross lives in a category 1 house and wants to insure his contents for £30 000 and the building for £230 000. How much will his monthly premium be?

$3 \times 5.40 + 23 \times 1.70 = £55.30$

9. 3D Shapes

9.1 Formulae

9.1 (a) I can work with formulae expressed in words.

You should be able to work with **formulae** expressed in words.

To change a number of kilometres to a number of miles: "Multiply the kilometres by five and divide the answer by eight". Change 32 km to miles. $32 \times 5 \div 8 = 20\ miles$	To change from degrees Fahrenheit (°F) to degrees Celsius (°C): "Subtract 32 from the temperature in °F, then divide the answer by 1·8". Change 86°F to °C. $86 - 32 \div 1.8 = 68.2^o C$	When buying pairs of jeans from a catalogue: "Multiply the number of jeans by 25 and add on 3·99". Work out the cost of 6 pairs of jeans. $6 \times 25 + 3.99 = £153.99$	The area of a quarter circle can be found: "Square the radius, multiply by 3·14 and divide by four". Find the area of a quarter circle with radius 3 cm. $10^2 \times 3.14 \div 4 = 78.5\ m^2$

9.1 (b) I can work with formulae expressed in symbols.

You can also evaluate a formula in context using **substitution.**

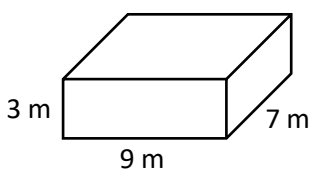
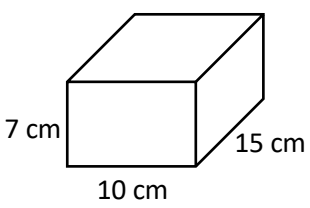
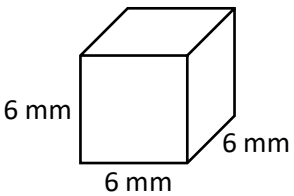
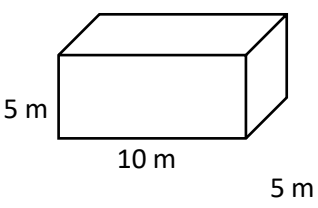
An approximate formula for converting celsius temperatures to fahrenheit is $F = 2C + 30$. Convert the temperature 15 degrees celsius into fahrenheit. $F = 2 \times 15 + 30$ $= 60^o F$	To calculate voltage (V) when you know the current (I) and resistance (R) the formula $V = IR$ is used. Find the voltage if the current is 30 volts and the resistance is 20 ohms. $V = 30 \times 20$ $= 600\ volts$	The volume of a cube is found by using the formulae $V = L^3$, where L is the length of a side of the cube. Calculate the volume of a cube which has a side of length 6 cm. $V = 6^3$ $= 216\ cm^3$	Joe's pay is calculated using $P = 5.5H + 9M$ where P is his pay (in £), H is the hours he works, and M is the number of phones he sells. Find his pay if he works 20 hours and sells 7 phones. $P = 5.5 \times 20 + 9 \times 7$ $= £173$

9.2 Volume

9.2 (a) I can calculate the volume of a cube or a cuboid.

The **volume** of cubes and cuboids can be founding using the formulae:

$$V_{cuboid} = L \times B \times H$$

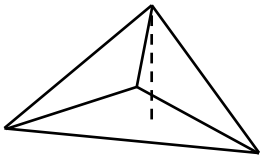
Calculate the volume.  $V_{cuboid} = 9 \times 7 \times 3$ $= 189\ m^3$	Calculate the volume.  $V_{cuboid} = 15 \times 10 \times 7$ $= 1050\ m^3$	Calculate the volume.  $V_{cuboid} = 6 \times 6 \times 6$ $= 216\ m^3$	Calculate the volume.  $V_{cuboid} = 10 \times 5 \times 5$ $= 250\ m^3$

9.2 (b) I can calculate the volume of basic three-dimensional shapes.

You should be able to calculate the **volume** of the following shapes. The latter three are given in the **exam formulae sheet**.

$V_{pyramid} = \frac{1}{3} \times A \times h$	$V_{cylinder} = \pi \times r^2 \times h$	$V_{cone} = \frac{1}{3} \times \pi \times r^2 \times h$	$V_{sphere} = \frac{4}{3} \times \pi \times r^3$

A pyramid has a base area 24 cm^2 and height 7 cm.

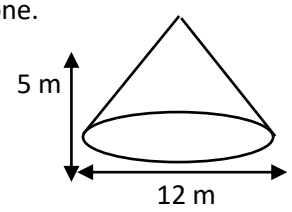


Find its volume.

$V_{pyramid} = \frac{1}{3} \times 24 \times 7$

$= 56\ cm^3$

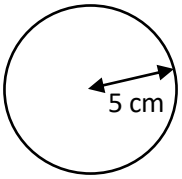
Find the volume of the cone.



$V_{cone} = \frac{1}{3} \times \pi \times 6^2 \times 5$

$= 188.5\ m^3$

Find the volume of the sphere.



$V_{sphere} = \frac{4}{3} \times \pi \times 5^3$

$= 523.6\ cm^3$

Find the volume of a cylinder with radius 5 metres and height 4 metres.

Do not use a calculator and instead use $\pi = 3.14$.

$V_{cylinder} = 3.14 \times 5^2 \times 4$

$= 3.14 \times 100$

$= 314\ cm^3$

9.2 (c) I can use the volume of a shape to calculate its height.

In some questions you may be given the **volume** of a **shape** and asked to calculate its **height**.

A cylinder has a volume of 2 300 m^3. The radius of the cross section is 8.2 m.

Calculate its' height.

$V_{cylinder} = \pi \times 8.2^2 \times h$

$2\ 300 = 211.24 \times h$

$h = 10.89\ m$

A cone has a volume of 127 cm^3. The radius of the cross section is 4 cm.

Calculate its' height.

$V_{cone} = \frac{1}{3} \times \pi \times 4^2 \times h$

$127 = 16.76 \times h$

$h = 7.58\ cm$

A square based pyramid has volume 324 m^3. The sides of the base are 9 m.

Calculate its' height.

$V_{pyramid} = \frac{1}{3} \times 81 \times h$

$324 = 27 \times h$

$h = 12\ m$

A cone has a volume of 5 890 cm^3. The radius of the cross section is 9 cm.

Calculate its' height.

$V_{cone} = \frac{1}{3} \times \pi \times 9^2 \times h$

$5\ 890 = 84.82 \times h$

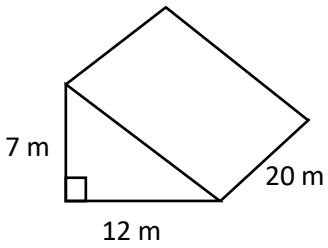
$h = 69.44\ cm$

9.2 (d) I can calculate the volume of a prism.

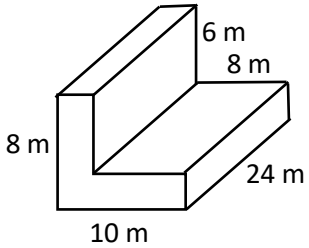
A **prism** is a solid that no matter where it is sliced along its length, the cross section is the same size and shape. The **volume** of a prism where A is the area of the cross-section and h is the height is given by:

$$V_{prism} = Ah$$

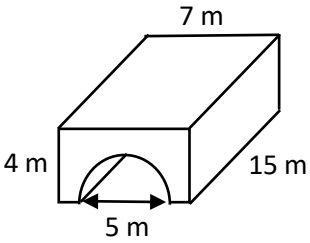
Find the volume.



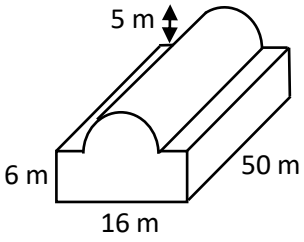
Find the volume.



Find the volume.



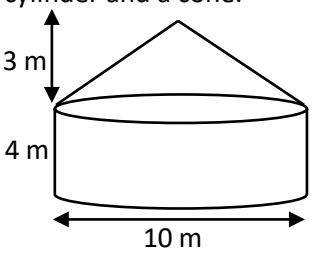
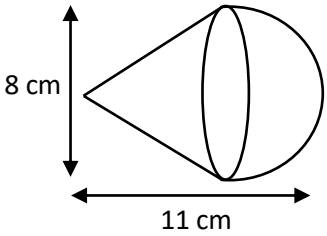
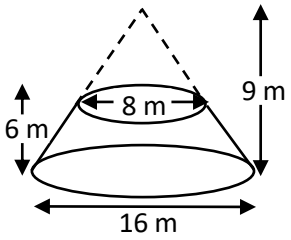
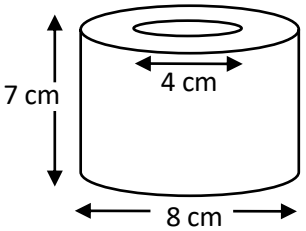
Find the volume.



$A_{triangle} = \frac{1}{2} \times 12 \times 7$ $= 42\ m^2$ $V_{prism} = 42 \times 20$ $= 840\ m^3$	$A_{rectangle} = 10 \times 2$ $= 20\ m^2$ $A_{rectangle} = 2 \times 6$ $= 12\ m^2$ $A_{total} = 20 + 12$ $= 32\ m^2$ $V_{prism} = 32 \times 24$ $= 768\ m^3$	$A_{rectangle} = 7 \times 4$ $= 28\ m^2$ $A_{s.\ circle} = \pi \times 2.5^2 \div 2$ $= 9.82\ m^2$ $A_{total} = 28 - 9.82$ $= 18.18\ m^2$ $V_{prism} = 18.18 \times 15$ $= 272.7\ m^3$	$A_{rectangle} = 16 \times 6$ $= 96\ m^2$ $A_{s.\ circle} = \pi \times 5^2 \div 2$ $= 39.27\ m^2$ $A_{total} = 96 + 39.27$ $= 135.27\ m^2$ $V_{prism} = 135.27 \times 50$ $= 6\ 763.5\ m^3$

9.2 (e) I can calculate the volume of composite shapes.

A **composite shape** is one made up of two or more standard shapes. You can calculate the volume of these.

A shape is made from a cylinder and a cone.  Calculate the volume of the shape, correct to **two significant figures**. $V_{cone} = \frac{1}{3} \times \pi \times 5^2 \times 3$ $= 78.54\ m^3$ $V_{cylinder} = \pi \times 5^2 \times 4$ $= 314.16\ m^3$ $V_{total} = 78.54 + 314.16$ $= 392.70$ $= 390\ m^3$	A shape is made from a hemisphere and a cone.  Calculate the volume of the shape, correct to **one significant figure**. $V_{cone} = \frac{1}{3} \times \pi \times 4^2 \times 7$ $= 117.29\ cm^3$ $V_{hemisphere} = \frac{2}{3} \times \pi \times 4^3$ $= 134.04\ cm^3$ $V_{total} = 117.29 + 134.04$ $= 251.33$ $= 300\ cm^3$	A shape is made from a cone with its top cut off.  Calculate the volume of the shape, correct to **one decimal place.** $V_{l\ cone} = \frac{1}{3} \times \pi \times 8^2 \times 9$ $= 603.19\ m^3$ $V_{s\ cone} = \frac{1}{3} \times \pi \times 4^2 \times 3$ $= 50.27\ m^3$ $V_{total} = 603.19 - 50.27$ $= 552.92$ $= 552.9\ m^3$	The following shape has had a section removed.  Calculate the volume of the shape, correct to **two significant figures**. $V_{l\ cylinder} = \pi \times 4^2 \times 7$ $= 351.86\ cm^3$ $V_{s\ cylinder} = \pi \times 2^2 \times 7$ $= 87.96\ m^3$ $V_{total} = 351.86 - 87.96$ $= 263.90$ $= 260\ m^3$

9.3 Packing Problems

9.3 (a) I can solve problems involving packing containers.

Problems involving **space** and **volume** occur often in everyday life.

The most common is in **packing problems**.

Cartons of Apple Juice measure 7 cm by 5 cm and are 20 cm tall. The cartons are packed into a box measuring 36 cm by 48 cm and 75 cm tall. What is the greatest number of cartons that can be packed into this cardboard box if the cartons must be stored upright?

$Option\ 1$	$36 \div 7 = 5$	$48 \div 5 = 9$	$75 \div 20 = 3$	$5 \times 9 \times 3 = 135$	$135\ is\ the\ maximum.$
$Option\ 2$	$36 \div 5 = 7$	$48 \div 7 = 6$	$75 \div 20 = 3$	$7 \times 6 \times 3 = 126$	

A truck delivers boxes of champagne. Each box is 60 cm by 50 cm by 40 cm high. The truck has a storage space measuring 3.5 m by 2.5 m and 2.2 m high. What is the maximum number of boxes that can be transported if the boxes are to be kept upright?

$Option\ 1$	$350 \div 60 = 5$	$250 \div 50 = 5$	$220 \div 40 = 5$	$5 \times 5 \times 5 = 125$	$140\ is\ the\ maximum.$
$Option\ 2$	$350 \div 50 = 7$	$250 \div 60 = 4$	$220 \div 40 = 5$	$7 \times 4 \times 5 = 140$	

9.3 (b) I can solve harder problems involving packing containers.

You should be able to solve more complex problems on this.

A factory warehouse is used to store large metal beams. The area designated to store these beams is 20 m by 10 m by 3 m. What is the maximum number of beams that could be stored if each one measures 3 m by 50 cm by 40 cm?

$Option\ 1$	$2\,000 \div 300 = 6$	$1\,000 \div 50 = 20$	$300 \div 40 = 7$	$6 \times 20 \times 7 = 840$
$Option\ 2$	$2\,000 \div 40 = 50$	$1\,000 \div 300 = 3$	$300 \div 50 = 6$	$50 \times 3 \times 6 = 900$
$Option\ 3$	$2\,000 \div 50 = 40$	$1\,000 \div 40 = 25$	$300 \div 300 = 1$	$40 \times 25 \times 1 = 1\,000$

$1\,000\ is\ the\ maximum.$

Large match boxes have dimensions of 6 cm by 15 cm by 20 cm. These boxes are packed into a cardboard box 180 cm by 100 cm by 60 cm. What is the maximum number of matchboxes that can be packed into the cardboard box?

$Option\ 1$	$180 \div 6 = 30$	$100 \div 15 = 6$	$60 \div 20 = 3$	$30 \times 6 \times 3 = 540$
$Option\ 2$	$180 \div 20 = 9$	$100 \div 6 = 16$	$60 \div 15 = 4$	$9 \times 16 \times 4 = 576$
$Option\ 3$	$180 \div 15 = 12$	$100 \div 20 = 5$	$60 \div 6 = 10$	$12 \times 5 \times 10 = 600$

$600\ is\ the\ maximum.$

10. Measures of Spread

10.1 Tolerance

10.1 (a) I can work with tolerance notation and intervals.

Tolerance is the **acceptable range of values** in which a measurement will be acceptable.

A wall is built using 30 bricks 186 ± 3 mm long. What is the maximum length of the wall?

186 ± 3 *has maximum value* 189.

$189 \times 30 = 5\,670$ *mm*

The maximum length of the wall is $5\,760$ *mm.*

A tower is made using 90 Pringles 2 ± 0.2 mm high. What is the minimum height of the tower?

2 ± 0.2 *has minimum value* 1.8.

$1.8 \times 90 = 162$ *mm*

The minimum height of the tower is 162 *mm.*

Jan makes candles in her home. These candles should be 20 cm tall. She rejects any candle outwith the range of ± 4 mm of this height.

20.2, 19.7, 20.5, 20.2, 20.0, 19.6, 19.5, 20.4, 19.7, 20.7

Calculate the percentage rejected from the sample.

20 ± 0.4 *has minimum value* 19.6 *and maximum value* 20.4.

$\frac{3}{10} = 30\%$ *of the candles will be rejected.*

A batch of pins is ordered with heights of 8.2 ± 0.3 mm. For the batch to be accepted, at least 80% of the pins in this sample must be within tolerance.

8.3, 7.9, 8.0, 8.3, 8.2, 7.7, 8.2, 7.6, 8.5, 7.8, 8.1, 7.3, 8.4, 7.9, 8.5, 8.3, 7.9, 8.2, 8.6, 8.0

Will this be accepted?

8.2 ± 0.3 *has minimum value* 7.9 *and maximum value* 8.5.

$\frac{15}{20} = 75\%$ *of the pins will be accepted.*

The bag will be rejected as only 75% *of the pins are acceptable.*

Packets of smarties should contain 45 smarties. Any packet outwith the range of ± 2 will be rejected.

44, 45, 45, 42, 46, 45, 47, 43, 48, 42, 43, 43, 46, 45, 45, 44, 47, 45

Calculate the percentage that will be accepted from the sample above.

45 ± 2 *has minimum value* 43 *and maximum value* 47.

$\frac{15}{18} = 83.33\%$ *of the packets will be accepted.*

10.1 (b) I can work with tolerance notation involving percentages.

You should also be able to work with **tolerance** and **percentages**.

A shop rejects pies if they do not weigh 200 g ± 2%.

202, 195, 204, 198, 200, 199, 203, 200, 203, 197, 198, 206

Calculate the fraction rejected.

1% *of* $200 = 4$

2% *of* $400 = 2 \times 2 = 4$

200 ± 4 *has minimum value* 196 *and maximum value* 204.

$\frac{2}{12} = \frac{1}{6}$ *pies will be rejected.*

A company will accept micro-chips if they weigh 40 g ± 5%.

43, 40, 38, 38, 39, 41, 37, 45, 39, 38, 41, 40, 40, 38

Calculate the fraction accepted.

1% *of* $40 = 0.4$

5% *of* $400 = 5 \times 0.4$

$= 2$

40 ± 2 *has minimum value* 38 *and maximum value* 42.

$\frac{3}{14}$ *miro chips will be accepted.*

Packets of crisps are rejected if they do not contain 40 ± 10%.

42, 46, 44, 38, 35, 39, 43, 41, 40, 42, 38, 35

Calculate the percentage rejected.

10% *of* $40 = 4$

40 ± 4 *has minimum value* 36 *and maximum value* 44.

$\frac{3}{12} = \frac{1}{4} = 25\%$ *of the packets will be rejected.*

A company accepts chocolate eggs if they weigh 200 g ± 2%.

200, 198, 202, 205, 204, 199, 201, 198, 200, 205

Calculate the percentage accepted.

1% *of* $200 = 2$

2% *of* $200 = 2 \times 2 = 4$

200 ± 4 *has minimum value* 196 *and maximum value* 204.

$\frac{8}{10} = 80\%$ *of the eggs will be accepted.*

10.2 Averages

10.2 (a) I can find the mean, median, mode and range of a data set.

An **average** indicates the **typical value** of a set of data. There are **three types** of **average**.

The **Mean**	Add the numbers together and divide the total by the amount of numbers.
The **Median**	1. Put the numbers in numerical order. 2. If the number of results is: (a) Odd, take the middle number. (b) Even, take the number halfway between the middle numbers.
The **Mode**	Take the number(s) which occurs most often in the set of data.

The **Range** is the difference between the **highest** and **lowest** numbers in a set of data.

Calculate the mean mode, median and range of:

4, 6, 14, 10, 6.

$Mean = \frac{4 + \cdots + 6}{5}$

$= \frac{40}{5} = 8$

$Mode = 6$

$4 \quad 6 \quad (6) \quad 10 \quad 14$

$Median = 6$

$Range = 14 - 4 = 10$

Calculate the mean mode, median and range of:

2, 5, 3, 5, 4, 11.

$Mean = \frac{2 + \cdots + 11}{6}$

$= \frac{30}{6} = 5$

$Mode = 5$

$2 \quad 3 \quad 4 \mid 5 \quad 5 \quad 11$

$Median = 4.5$

$Range = 11 - 2 = 9$

Calculate the mean mode, median and range of:

10, 9, 7, 20.

$Mean = \frac{10 + \cdots + 20}{4}$

$= \frac{46}{4} = 11.5$

$Mode = 7, 9, 10, 20$

$7 \quad 9 \mid 10 \quad 20$

$Median = 9.5$

$Range = 20 - 7 = 13$

Calculate the mean mode, median and range of:

3, 5, 6, 6, 7, 9, 13.

$Mean = \frac{3 + \cdots + 13}{7}$

$= \frac{49}{7} = 7$

$Mode = 6$

$3 \quad 5 \quad 6 \quad (6) \quad 7 \quad 9 \quad 13$

$Median = 6$

$Range = 13 - 3 = 10$

10.3 Box Plots

10.3 (a) I can find the quartiles of a data set.

You are expected to work with **quartiles** -these divide an **ordered list** into **four equally-sized parts.**

5, 2, 1, 7, 4, 1, 1.

$1 \quad (1) \quad 1 \quad (2) \quad 4 \quad (5) \quad 7$

$Q_1 \quad Q_2 \quad Q_3$

42, 50, 63, 55, 49.

$42 \mid 49 \quad (50) \quad 55 \mid 63$

$45.5 \quad Q_2 \quad 59$

$Q_1 \quad Q_3$

22, 27, 23, 29, 35, 18.

$18 \quad (22) \quad 23 \mid 27 \quad (29) \quad 35$

$Q_1 \quad 25 \quad Q_3$

Q_2

4, 8, 3, 4, 5, 8, 1, 9.

$1 \quad 3 \mid 4 \quad 4 \mid 5 \quad 8 \mid 8 \quad 9$

$3.5 \quad 4.5 \quad 8.5$

$Q_1 \quad Q_2 \quad Q_3$

10.3 (b) I can find the interquartile and semi-interquartile ranges.

You should also be able to calculate **interquartile** and **semi-interquartile ranges**:

- The **Interquartile range** is the difference between the upper and lower quartiles: $IQR = Q3 - Q1$
- The **Semi-Interquartile range** is half of this: $SIQR = (Q3 - Q1) \div 2$

Find the IQR of:	Find the SIQR of:	Find the SIQR of:	If $Q_1 = 12$, $Q_2 = 14$ and $Q_3 = 18$, find the IQR and SIQR.
4, 3, 8, 7, 2, 9. $2\ (3)\ 4 \mid 7\ (8)\ 9$ $Q_1 \quad 5.5 \quad Q_3$ Q_2 $IQR = 8 - 3$ $= 5$	9, 7, 6, 9, 8, 4, 5. $4\ (5)\ 6\ (7)\ 8\ (9)\ 9$ $Q_1 \quad Q_2 \quad Q_3$ $SIQR = (9 - 5) \div 2$ $= 2$	1, 4, 2, 4, 7, 9, 3, 8. $1\ \ 2 \mid 3\ \ 4 \mid 4\ \ 7 \mid 8\ \ 9$ $2.5 \quad 4 \quad 7.5$ $Q_1 \quad Q_2 \quad Q_3$ $SIQR = (7.5 - 2.5) \div 2$ $= 2.5$	$IQR = 18 - 12$ $= 6$ $SIQR = 6 \div 2$ $= 3$

10.3 (c) I can compare the median and interquartile ranges of data sets.

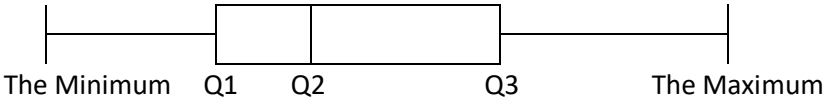
When comparing the **median** and **interquartile ranges** of two data sets:

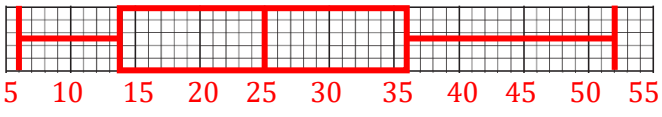
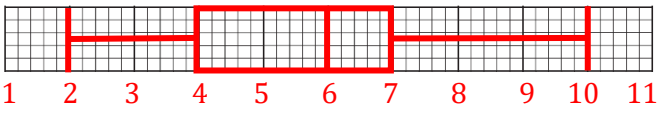
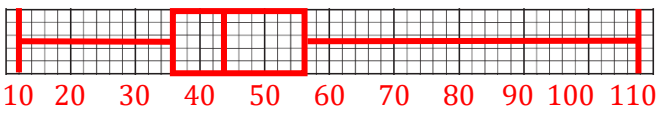
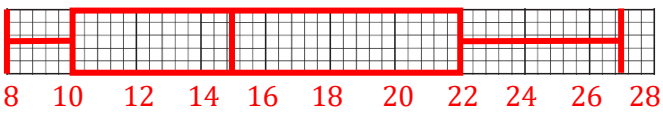
- Use the expression '**on average**' instead of **median.**
- A **larger interquartile range** means '**more varied**' or '**less consistent**'.
- **Never** use both **varied** and **consistent** in the same example.

In January the median temperature in Mexico City is 22°C and the interquartile range is 4.5. In Toronto meanwhile they are 2°C and 6. Compare the cities' temperatures. *On average the temperature in Mexico city is higher.* *The temperatures in Toronto are more varied.*	In their spelling test a P2 class had a median score of 6 and an interquartile range of 3. For the P3 class these were 8 and 4 respectively. Compare the class' performance in the tests. *On average the test scores were better in the P3 class.* *The number of marks was more varied in the P3 class.*	In Alness the median rent is £420 and the interquartile range is £160. In Invergordon these are £420 and £102 respectively. Compare the class' performance in the tests. *On average the rent is the same in both towns.* *The rent is more varied in Alness.*	The median number of hours that pupils had spent studying for a test was 12 and the interquartle range was 6. The next year they were 12 and 4. Comment on these. *On average the number of hours spent revising was the same.* *The number of hours spent revising by the pupils was more varied the first year.*

10.3 (d) I can construct a box plot to represent a data set.

A **box plot** splits a set of data into 4 sections (25% of the data in each).



Illustrate the following with a box plot: L = 6, Q1 = 14, Q2 = 25, Q3 = 36, H = 52.  	Illustrate the following with a box plot: L = 2, Q1 = 4, Q2 = 6, Q3 = 7, H = 10. 
Illustrate the following with a box plot: L = 12, Q1 = 36, Q2 = 44, Q3 = 56, H = 108.  	Illustrate the following with a box plot: L = 8, Q1 = 10, Q2 = 15, Q3 = 22, H = 27. 

10.4 Standard Deviation

10.4 (a) I can calculate the standard deviation of a data set.

The **standard deviation** of a set of numbers is a measure of how spread out they are from the **mean**. It is calculated using the following formula given in the **exam formulae sheet**:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

where $\bar{x}$ is the **mean**, n is the **number of numbers** and $\sum(x-\bar{x})^2$ is the **sum of the differences** between the set of numbers and the mean.

Calculate the mean and standard deviation of 2, 3, 9, 6, 5.

$$\bar{x} = \frac{2+\cdots+5}{5} = \frac{25}{5} = 5$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	-3	9
3	-2	4
5	0	0
6	1	1
9	4	16
25	0	30

$$s = \sqrt{\frac{30}{5-1}} = 2.74$$

Calculate the mean and standard deviation 21, 17, 14, 24, 19.

$$\bar{x} = \frac{21+\cdots+19}{5} = \frac{95}{5} = 19$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
21	2	4
17	-2	4
14	-5	25
24	5	25
19	0	0
95	0	58

$$s = \sqrt{\frac{58}{5-1}} = 3.81$$

Calculate the mean and standard deviation 4, 10, 8, 7, 3, 4.

$$\bar{x} = \frac{4+\cdots+4}{6} = \frac{36}{6} = 6$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
4	-2	4
10	4	16
8	2	4
7	1	1
3	-3	9
4	-2	4
36	0	38

$$s = \sqrt{\frac{38}{6-1}} = 2.76$$

10.4 (b) I can compare the mean and standard deviations of two data sets.

When comparing the **mean** and **standard deviation** of two data sets:

- Use the expression '**on average**' instead of **mean.**
- A **larger standard deviation** means '**more varied**' or '**less consistent**'.
- **Never** use both **varied** and **consistent** in the same example.

In March the mean temperature in Inverness is 7°C and the standard deviation is 2.5. In London meanwhile they are 11°C and 3 respectively.

Compare the cities' temperatures.

On average the temperature in London is higher.

The temperatures in London are more varied.

The mean number of goals scored by Dundee was 3.2 per game and for Falkirk 2.8. The standard deviation for Dundee was 0.9 and for Falkirk 1.3.

Compare the number of goals scored by the clubs.

On average the number of goals scored a game by Dundee is higher.

The number of goals scored by Falkirk was more varied.

In their Unit One test a class had a mean score of 58 and a standard deviation of 11.2. In their Unit Two test these were 55 and 7.6 respectively.

Compare the class' performance in the tests.

On average the test scores were better in the Unit One test.

The number of marks was more varied in the Unit One test.

The mean number of hours that pupils had spent studying for a test was 15 and the standard deviation was 3.4.

The next year they were 15 and 4.2. Comment on these.

On average the number of hours spent revising was the same.

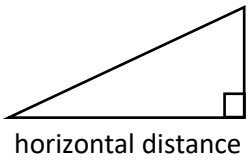
The number of hours spent revising by the pupils was more varied the second year.

11. Straight Lines

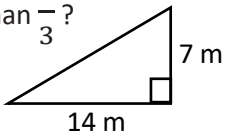
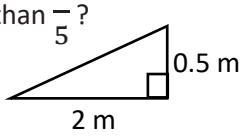
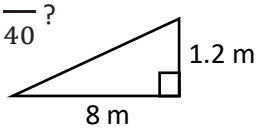
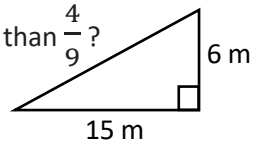
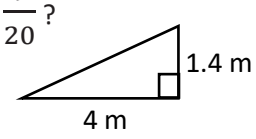
11.1 Gradient

11.1 (a) I can identify the gradient of a straight line as a fraction.

Gradient is a measure of how **steep** a **slope** is. If a gradient is **positive** it slopes **up**. If it is **negative** it slopes **down**. The following is given in the **formulae sheet**.

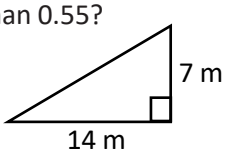
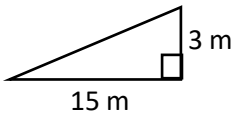
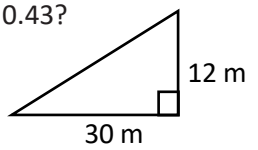
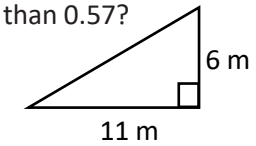
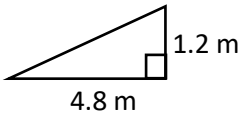


$$\text{Gradient} = \frac{\text{vertical height}}{\text{horizontal distance}}$$

Is the following gradient greater than $\frac{1}{3}$?  $Gradient = \frac{7}{14}$ $= \frac{1}{2}$ $Yes\ as\ \frac{1}{2} > \frac{1}{3}.$	Is the following gradient greater than $\frac{1}{5}$?  $Gradient = \frac{0.5}{2}$ $= \frac{1}{4}$ $Yes\ as\ \frac{1}{4} > \frac{1}{5}.$	Is the following gradient less than $\frac{7}{40}$?  $Gradient = \frac{1.2}{8}$ $= \frac{12}{80}$ $= \frac{6}{40}$ $Yes\ as\ \frac{6}{40} < \frac{7}{40}.$	Is the following gradient greater than $\frac{4}{9}$?  $Gradient = \frac{6}{15}$ $= \frac{2}{5}$ $= \frac{4}{10}$ $No\ as\ \frac{4}{10} < \frac{4}{9}.$	Is the following gradient less than $\frac{9}{20}$?  $Gradient = \frac{1.4}{4}$ $= \frac{14}{40}$ $= \frac{7}{20}$ $Yes\ as\ \frac{7}{20} < \frac{9}{20}.$

11.1 (b) I can identify the gradient of a straight line as a decimal.

You can also express a **gradient** as a **decimal**.

Is the following gradient greater than 0.55?  $Gradient = \frac{9}{18}$ $= \frac{1}{2}$ $= 0.5$ $No\ as\ 0.5 < 0.55.$	Is the following gradient greater than 0.18?  $Gradient = \frac{3}{15}$ $= \frac{1}{5}$ $= 0.2$ $Yes\ as\ 0.2 > 0.18.$	Is the following gradient less than 0.43?  $Gradient = \frac{12}{30}$ $= \frac{2}{5}$ $= 0.4$ $Yes\ as\ 0.4 < 0.43.$	Is the following gradient greater than 0.57?  $Gradient = \frac{6}{11}$ $= 0.545$ $No\ as$ $0.545 < 0.57.$	Is the following gradient less than 0.22?  $Gradient = \frac{1.2}{4.8}$ $= \frac{12}{48}$ $= \frac{1}{4}$ $= 0.25$ $No\ as\ 0.25 > 0.22.$

11.1 (c) I can identify the gradient of a line from its coordinates.

You should also be able to identify the **gradient** of a line from its **coordinates**.

Find the gradient of the straight line passing through (2, -3) and (8, -7).	Find the gradient of the straight line passing through (5, 0) and (1, -8).	Find the gradient of the straight line passing through (6, -1) and (-2, -13).	Find the gradient of the straight line passing through (5, -2) and (10, -7).	Find the gradient of the straight line passing through (4, 9) and (-5, -3).
$Gradient = -\frac{4}{6}$ $= -\frac{2}{3}$	$Gradient = \frac{8}{4}$ $= 2$	$Gradient = \frac{12}{8}$ $= \frac{3}{2}$	$Gradient = -\frac{5}{5}$ $= -1$	$Gradient = \frac{12}{9}$ $= \frac{4}{3}$

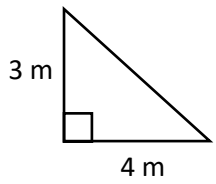
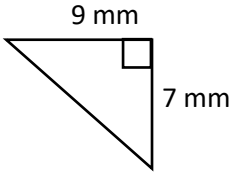
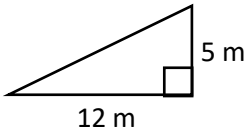
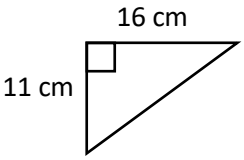
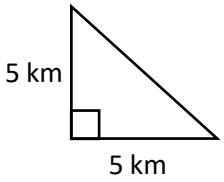
11.2 Pythagoras Theorem

11.2 (a) I can use Pythagoras' theorem to calculate the hypotenuse.

Pythagoras' theorem states that for all **right-angled triangles**, 'The square of the **hypotenuse** (the longest side) is equal to the sum of the squares on the other two sides.'

It can be calculated using the formula below given in the **formula sheet**:

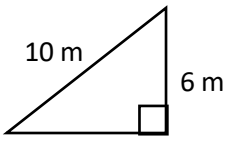
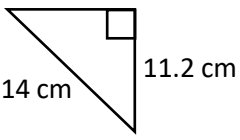
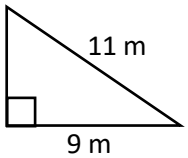
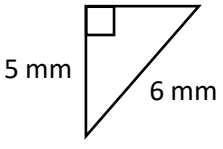
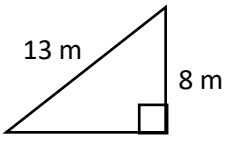
$$c^2 = a^2 + b^2$$

				
$c^2 = 3^2 + 4^2$ $= 25$ $c = \sqrt{25}$ $= 5\ m$	$c^2 = 9^2 + 7^2$ $= 130$ $c = \sqrt{130}$ $= 11.40\ mm$	$c^2 = 12^2 + 5^2$ $= 169$ $c = \sqrt{169}$ $= 13\ m$	$c^2 = 11^2 + 16^2$ $= 377$ $c = \sqrt{377}$ $= 19.42\ cm$	$c^2 = 5^2 + 5^2$ $= 50$ $c = \sqrt{50}$ $= 7.07\ km$

11.2 (b) I can use Pythagoras' theorem to calculate a shorter side.

Rearranging this formula allows you to calculate one of the **shorter sides** using:

$$b^2 = c^2 - a^2$$

				
$b^2 = 10^2 - 6^2$ $= 64$ $b = \sqrt{64}$ $= 8\ m$	$b^2 = 14^2 - 11.2^2$ $= 70.56$ $b = \sqrt{70.56}$ $= 8.4\ cm$	$b^2 = 11^2 - 9^2$ $= 40$ $b = \sqrt{40}$ $= 6.32\ m$	$b^2 = 6^2 - 5^2$ $= 11$ $b = \sqrt{11}$ $= 3.32\ mm$	$b^2 = 13^2 - 8^2$ $= 105$ $b = \sqrt{105}$ $= 10.25\ m$

12. Pay Slips

12.1 Gross Pay

12.1 (a) I can carry out calculations based on wages and salaries.

You should be able to perform calculations based on **wages** and **salaries**.

Ed is paid £10·76 per hour. How much will he get for a 40-hour week? $40 \times 10.76 = £430.40$	Ted is paid £641·70 for 45 hours of work. Find his hourly rate. $641.70 \div 45 = £14.26$	Simon is paid £36 048 per annum. What is his monthly pay? $36\,048 \div 12 = £3\,004$	Pila earns £317·10 per week. What are her yearly earnings? $52 \times 317.10 = £16\,489.20$

12.1 (b) I can calculate a wage involving overtime pay.

Overtime is extra work above the normal numbers of work. It is usually paid at a higher rate of pay.

Time and a Half	1.5 x the normal rate of pay
Double Time	2 x the normal rate of pay

Claire is paid £7.50 an hour and is contracted to work 40 hours a week. Overtime pay is paid at time and a half. How much will she earn in a week where she works a total of 48 hours? $40 \times 7.50 + 8 \times 1.5 \times 7.50 = £390$	Josh earns £8.30 an hour as a plumber's mate. He also gets paid double time on a Saturday. How much would he be paid for working 38 hours from Monday to Friday and 4 hours on Saturday? $38 \times 8.30 + 4 \times 2 \times 8.30 = £381.80$	Mark earns £9 an hour and works 9 am to 4 pm Monday to Friday. He is paid time and a half on Saturdays and double time on Sundays and works 3 hours on each. Calculate his total wage. $From\ 9\ am\ to\ 4\ pm\ there\ is\ 7\ hours.$ $35 \times 9 + 3 \times 1.5 \times 9 + 3 \times 2 \times 9 = £409.50$	Jan is paid £11.40 an hour for his normal 40-hour work week and double time for hours above this. If he earns £524.40 one week then calculate how many hours of overtime he must have worked. $40 \times 11.40 = £456$ $524.40 - 456 = £68.40$ $68.40 \div 11.40 \div 2 = 3\ hours\ overtime$

12.1 (c) I can calculate a wage involving commission.

Employees who sell things in their job may earn **commission** as part of their salary. This is usually calculated as a **percentage** of the value of their sales.

Tam sells used cars. He is paid a commission of 5% on any car he sells. Last week, Tam sold £9 000 worth of cars. How much commission would Tam earn? $0.05 \times 9\,000 = £450$	Laura has a basic pay of £12 000 per annum. She also earns 8% commission on sales over £10 000. Last year, Laura sold £35 000 worth of clothes. What is she paid for this? $35\,000 - 10\,000 = £25\,000$ $12\,000 + 0.08 \times 25\,000 = £32\,000$	Ann sells kitchens. For every home visit she is paid £12. For every sale she gets 1% commission. 43 customers visit and there are £42 000 in sales. Calculate Ann's pay. $43 \times 12 + 0.01 \times 42\,000 = £936$	Colin has a basic pay of £42 000 per annum. He also earns 1% commission on sales over £20 000. Last year, Colin sold £60 000 worth of items. What is his pay? $42\,000 - 20\,000 = £22\,000$ $42\,000 + 0.01 \times 22\,000 = £42\,220$

12.2 Net Pay

12.2 (a) I can calculate net pay given gross pay and deductions.

Gross Pay is the amount of money an employer pays you.

Deductions are taken from your gross pay and include things like:

- **Superannuation** – private pension for when you retire
- **National Insurance** (NI) – pays for loss of earnings of you are sick / unemployed
- **Income Tax** – tax paid top the government to pay for education, health, transport etc.

Net pay is the amount you **take home** after deductions are made.

Jack has an annual salary of £15 200. He has to pay Superannuation of £920, National Insurance of £1 200 and Income Tax of £1 152. Calculate Jack's annual net pay. $G.P: £15\,200$ $D: 920 + 1\,200 + 1\,152$ $= £3\,272$ $N.P: 15\,200 - 3\,272$ $= £11\,928$	Ethan is paid £620 a week. He has to pay £91 Superannuation, £76 National Insurance and Income Tax of 5% his gross wage. Calculate how much Ethan's weekly net pay is. $G.P: £635$ $D: 91 + 76 + 0.05 \times 635$ $= £198$ $N.P: 635 - 198 = £437$	Clara has a monthly salary of £2 430. She has to pay Superannuation of £127, National Insurance of £164 and Income Tax of £152. Calculate Jack's annual net pay. $G.P: £2\,430$ $D: 127 + 164 + 152$ $= £443$ $N.P: 2\,430 - 443$ $= £1\,987$	Amir has an annual salary of £42 300. He has to pay Superannuation of £3 520, National Insurance of £4 270 and Income Tax of £3 952. Calculate Amir's annual net pay. $G.P: £42\,300$ $D: 3\,520 + 4\,270$ $+3\,952 = £11\,742$ $N.P: 42\,300 - 11\,742$ $= £30\,558$

12.2 (b) I can solve problems involving net pay.

You should be able to solve problems based on **net pay**.

For Katie's new job she is paid an annual net salary of £64 000. Her deductions total up to £15 275 per annum. Calculate Katie's annual gross pay. $N.P: £64\,000$ $D: £15\,275$ $G.P: 64\,000 + 15\,275$ $= £79\,275$	For Hugh's new job his net pay is £42 400 and his gross pay is £56 700. Of his deductions Hugh's income tax makes up 30%. Calculate how much this is. $G.P: £56\,700$ $N.P: £42\,400$ $D: 56\,700 - 42\,400$ $= £14\,300$ $0.3 \times 14\,300 = £4\,290$	Dave is paid a monthly gross salary of £1 570. His deductions total £425 monthly. Dave is saving 20% of his monthly salary. How long will it take him to save up £6 000? $G.P: £1\,570$ $D: £425$ $N.P: 1\,570 - 425$ $= £1\,145$ $0.2 \times 1\,145 = £229$ $6\,000 \div 229 = 26.20$ $\approx 27\ months$	Hannah earns £56 450 net pay after she has paid Superannuation of £6 400, National Insurance of £4 670 and Income Tax of £3 850. Calculate her gross pay. $N.P: £56\,450$ $D: 6\,400 + 4\,670$ $+3\,850 = £14\,920$ $G.P: 56\,450 + 14\,920$ $= £71\,370$

12.3 Deductions

12.3 (a) I can calculate how much Income Tax a person should pay.

The **Inland Revenue** calculate your **income tax** not purely on your gross income. Instead, they give you allowances and relief on part of your income. Income is taxed at a standard rate of 20%.

Timothy earns £17 000. He has a £9 520 tax allowance. Calculate his income tax. $17\,000 - 9\,520 = £7\,480$ $0.2 \times 7\,480 = £1\,496$	Naomi earns £35 600. She has a £7 860 tax allowance. Calculate her income tax. $23\,650 - 8\,014 = £27\,740$ $0.2 \times 27\,740 = £5\,548$	Ryan earns £23 650. He has a tax allowance of £8 014. Calculate his income tax. $23\,650 - 8\,014 = £15\,636$ $0.2 \times 15\,636 = £3\,127.20$	Amy earns £28 400. She has a tax allowance of £10 566. Calculate her income tax. $28\,400 - 10\,566 = £17\,834$ $0.2 \times 15\,636 = £3\,556.80$

12.3 (b) I can calculate how much National Insurance a person should pay.

National Insurance contributions are a tax on earnings paid by employees and employers and help fund your entitlement to certain state benefits, such as the **State Pension** and **Maternity Allowance**. They are calculated using the table on the left.

National Insurance Rates	
Up to £9 500	0 %
From £9 500 to £50 000	12 %
More than £50 000	2 %

John earns £22 950 annually. Calculate how much National Insurance he pays. $22\,950 - 9\,500 = £13\,450$ $0.12 \times 13\,450 = £1\,614$	Natalie earns £51 600 annually. Calculate how much National Insurance she pays. $50\,000 - 9\,500 = £40\,500$ $0.12 \times 40\,500 = £4\,860$ $51\,600 - 50\,000 = £1\,600$ $0.02 \times 1\,600 = £32$ $4\,860 + 32 = £4\,892$	Patrick earns £63 400 annually. Calculate how much National Insurance he pays. $50\,000 - 9\,500 = £40\,500$ $0.12 \times 40\,500 = £4\,860$ $63\,400 - 50\,000 = £13\,400$ $0.02 \times 13\,400 = £268$ $4\,860 + 268 = £5\,128$	Saud earns £126 200 annually. Calculate how much National Insurance he pays. $50\,000 - 9\,500 = £40\,500$ $0.12 \times 40\,500 = £4\,860$ $126\,200 - 50\,000 = £76\,200$ $0.02 \times 76\,200 = £1\,524$ $4\,860 + 1\,524 = £6\,384$